Day 203 **IGCSE 10-a-day: Model Solutions**

1 Median and quartiles from a list

Here are 11 numbers.
55, 21, 16, 6, 27, 18, 17, 37, 36, 49, 11
Find (a) the median (b) the interquartile range.

Ordered: 6, 11, 16, 17, 18, 21, 27, 36, 37, 49, 55
(a) Median is the 6th value = 21
(b) Lower quartile is the 3rd value = 16; upper quartile is the 9th value = 37
IQR = 37 − 16 = 21
Answer: (a) 21 (b) 21

2 Tree diagrams without replacement

A bag contains 2 red beads, 5 blue beads and 2 green beads.
Two beads are taken at random without replacement.
Work out the probability that (a) they are different colours, (b) at least one is red.

(a) P(different) = 1 − P(same) = $1 - \frac{1}{3} = \frac{2}{3}$

(b) P(at least one red) = 1 − P(none) = $1 - \frac{7}{9} \times \frac{6}{8} = \frac{5}{12}$

Answer: (a) $\frac{2}{3}$ (b) $\frac{5}{12}$

3 Algebraic fractions: multiplying and dividing

Simplify fully $\frac{x^2 + 13x + 42}{x^2 - 5x} \times \frac{x^2 + x}{x + 6}$

Factorise: $\frac{(x + 6)(x + 7)}{x(x - 5)} \times \frac{x(x + 1)}{(x + 6)}$
Cancel (x + 6) and x
$= \frac{(x + 1)(x + 7)}{(x - 5)}$
Answer: $\frac{(x + 1)(x + 7)}{(x - 5)}$

4 Tangent to a curve with negative or fractional powers

A curve has equation $y = -x^2 - \frac{5}{x}$.

Find the equation of the tangent to the curve at the point where x = 4.

$\frac{dy}{dx} = -2x + 5x^{-2}$

At x = 4: $y = -\frac{69}{4}$ and gradient $= -\frac{123}{16}$

$y + \frac{69}{4} = -\frac{123}{16}(x - 4)$

$y = -\frac{123}{16}x + \frac{27}{2}$

Answer: $y = -\frac{123}{16}x + \frac{27}{2}$

5 Collinear points with vectors

$\mathbf{OA} = 3\mathbf{a}$, $\mathbf{OB} = -\mathbf{a} + 4\mathbf{b}$ and $\mathbf{OC} = 5\mathbf{a} - 2\mathbf{b}$.
Show that A, B and C lie on a straight line.

$\mathbf{AB} = \mathbf{OB} - \mathbf{OA} = -4\mathbf{a} + 4\mathbf{b}$
$\mathbf{AC} = \mathbf{OC} - \mathbf{OA} = 2\mathbf{a} - 2\mathbf{b}$

$\mathbf{AC} = -\frac{1}{2}\mathbf{AB}$

AC is a multiple of AB and they share the point A, so A, B and C are collinear

Answer: $\mathbf{AC} = -\frac{1}{2}\mathbf{AB}$, so A, B, C are collinear

6 Gradient function from a given point

A curve has equation $y = -2x^3 + 12x^2 - 18x + 2$.
The point A $(-1, k)$ lies on the curve. Work out the value of k and the gradient of the curve at A.

Substitute $x = -1$ into the equation of the curve
$y = 34$ when $x = -1$, so $k = 34$
$\frac{dy}{dx} = -6x^2 + 24x - 18$
At A: gradient $= -48$
Answer: k = 34, gradient = −48

7 Equations in disguise

Solve $x - 5\sqrt{x} + 6 = 0$

Let $u = \sqrt{x}$: $u^2 - 5u + 6 = 0$
Factorise: $(u - 3)(u - 2) = 0$, so $u = 3$ or $u = 2$
$\sqrt{x} = 3$ or $\sqrt{x} = 2$
$x = 9$ or $x = 4$
Answer: x = 4 or x = 9

8 Tangent to a circle

A circle has equation $x^2 + y^2 = 169$.
The point P(5, −12) lies on the circle.
(a) Find the equation of the tangent to the circle at P. Give your answer in the form ax + by = c, where a, b and c are integers.
(b) The tangent meets the x-axis at the point T. Write down the coordinates of T.

(a) The gradient of the radius OP is $-\frac{12}{5}$

The tangent is perpendicular to the radius, so its gradient is $-1 \div (-\frac{12}{5}) = \frac{5}{12}$

$y - (-12) = \frac{5}{12}(x - 5)$
Multiply out and rearrange: $5x - 12y = 169$
(b) On the x-axis $y = 0$, so $5x = 169$
$x = \frac{169}{5}$, so T is $(\frac{169}{5}, 0)$

Answer: (a) 5x − 12y = 169 (b) T = $(\frac{169}{5}, 0)$

9 Rationalising the denominator

Write $\frac{1 + \sqrt{5}}{3 + \sqrt{5}}$ in the form $p + q\sqrt{5}$
where p and q are rational numbers.

Multiply top and bottom by $(3 - \sqrt{5})$:
Numerator: $(1 + \sqrt{5})(3 - \sqrt{5}) = -2 + 2\sqrt{5}$
Denominator: $9 - 5 = 4$
$= \frac{-1 + \sqrt{5}}{2}$

Answer: $\frac{-1 + \sqrt{5}}{2}$

10 Open box of fixed surface area

An open box has a square base of side x cm and height h cm. The total outside surface area is 12 cm^2.

(a) Show that the volume of the box is $V = \frac{1}{4}(12x - x^3)$ cm^3.

(b) Find the maximum volume of the box, proving that it is a maximum.

$x^2 + 4xh = 12$, so $h = \frac{12 - x^2}{4x}$

$V = x^2h = \frac{x(12 - x^2)}{4} = \frac{1}{4}(12x - x^3)$

$\frac{dV}{dx} = -\frac{3}{4}x^2 + 3 = 0$, so $x^2 = 4$ and $x = 2$

$\frac{d^2V}{dx^2} = -\frac{3}{2}x = -3 < 0$, so a maximum

$V = 4$

Answer: 4 cm^3