Day 202 **IGCSE 10-a-day: Model Solutions**

1 Magnitude of a vector

$\mathbf{v} = \begin{pmatrix} 6 \\ -6 \end{pmatrix}$.

Find $|\mathbf{v}|$. Give your answer as a surd in its simplest form.

$|\mathbf{v}| = \sqrt{(6)^2 + (-6)^2} = \sqrt{36 + 36}$

$= \sqrt{72} = 6\sqrt{2}$

Answer: $6\sqrt{2}$

2 Prime factors and indices

Write $25^4 \times 12^2$ as a product of powers of prime numbers.

$25 = 5^2$, $12 = 2^2 \times 3$

$25^4 = 5^8$

$12^2 = 2^4 \times 3^2$

Add the powers of each prime: $2^4 \times 3^2 \times 5^8$

Answer: $2^4 \times 3^2 \times 5^8$

3 Averages with algebra

Here are five numbers: $3x$, $3x - 2$, $x + 6$, $3x + 1$, $3x + 3$.

The mean of the five numbers is 25.

(a) Work out the value of x.

(b) Work out the median of the five numbers.

(a) Total = $(3x) + (3x - 2) + (x + 6) + (3x + 1) + (3x + 3) = 25 \times 5 = 125$

$13x + 8 = 125$, so $13x = 117$ and $x = 9$

The numbers are 27, 25, 15, 28, 30

(b) In order: 15, 25, 27, 28, 30, so the median is 27

Answer: (a) x = 9 (b) 27

4 Rearrange then differentiate

$y = \dfrac{6x^3 + 5x}{x}$

Find $\dfrac{dy}{dx}$.

Divide each term by x: $y = 6x^2 + 5$

$\dfrac{dy}{dx} = 12x$

Answer: $\dfrac{dy}{dx} = 12x$

5 Iteration

Use the iteration formula $x_{n+1} = \sqrt{3x_n + 10}$ with $x_0 = 4$ (Calculator)

to work out x_1, x_2 and x_3.

Give each answer correct to 3 decimal places.

$x_1 = \sqrt{3 \times 4 + 10} = 4.69042...$

$x_2 = \sqrt{3 \times x_1 + 10} = 4.90625...$

$x_3 = \sqrt{3 \times x_2 + 10} = 4.97179...$

Rounded: $x_1 = 4.690$, $x_2 = 4.906$, $x_3 = 4.972$

Answer: $x_1 = 4.690$, $x_2 = 4.906$, $x_3 = 4.972$

6 Show that dy/dx = ...

$y = \frac{(x+4)^2}{x}$ $(x \neq 0)$

(a) Show that $\frac{dy}{dx} = \frac{(x-4)(x+4)}{x^2}$.

(b) Find the gradient when $x = 5$.

Expand: $(x+4)^2 = x^2 + 8x + 16$, so $y = x + 8 + 16x^{-1}$

$\frac{dy}{dx} = 1 - \frac{16}{x^2} = \frac{x^2 - 16}{x^2} = \frac{(x-4)(x+4)}{x^2}$

When $x = 5$: $\frac{dy}{dx} = \frac{9}{25}$

Answer: (b) $\frac{9}{25}$

7 Trigonometric graphs

The diagram shows the graph of $y = a\cos x° + c$ for $0 \leq x \leq 360$.
Some points on the graph are labelled.
Find the value of a and the value of c.

At $x = 0$, $\cos x° = 1$, so $a + c = -5$
At $x = 180$, $\cos x° = -1$, so $-a + c = -7$
Add: $2c = -12$, so $c = -6$
Subtract: $2a = 2$, so $a = 1$
Answer: $a = 1$, $c = -6$

8 Shaded areas: exact answers

ABCD is a square of side 10 cm.
BD is an arc of a circle with centre A and an arc of a circle with centre C.
Work out the area of the shaded region. Give your answer in terms of π.

Area of quarter circle (centre A, radius 10) $= \frac{1}{4} \times \pi \times 10^2 = 25\pi$

The two quarter circles overlap in the shaded region and together cover the whole square
Shaded area $= 2 \times 25\pi - 100 = 50\pi - 100$
Answer: $(50\pi - 100)$ cm^2

9 Tangent meets the curve again

A curve has equation $y = x^3 - x^2 - 6x + 1$.
The tangent to the curve at the point A, where $x = 0$, meets the curve again at the point B.
Find the coordinates of B.

$\frac{dy}{dx} = 3x^2 - 2x - 6$, so the gradient at A is -6, and A = (0, 1)
Tangent: $y = -6x + 1$
Equate: $x^3 - x^2 = 0$, which has a repeated root $x = 0$
Sum of the roots = 1, so $0 + 0 + x = 1$ and $x = 1$
$y = -5$
Answer: B (1, −5)

10 Finding a matrix from MA = B

$A = \begin{pmatrix} 2 & -1 \\ 4 & -3 \end{pmatrix}$ and $B = \begin{pmatrix} 2 & -1 \\ 4 & -5 \end{pmatrix}$.

AM = B, where M is a 2 × 2 matrix.

Find M.

AM = B so $M = A^{-1}B$

$A^{-1} = -\frac{1}{2}\begin{pmatrix} -3 & 1 \\ -4 & 2 \end{pmatrix}$

$M = -\frac{1}{2}\begin{pmatrix} -3 & 1 \\ -4 & 2 \end{pmatrix}\begin{pmatrix} 2 & -1 \\ 4 & -5 \end{pmatrix} = \begin{pmatrix} 1 & 1 \\ 0 & 3 \end{pmatrix}$

Answer: $\mathbf{M} = \begin{pmatrix} \mathbf{1} & \mathbf{1} \\ \mathbf{0} & \mathbf{3} \end{pmatrix}$