Day 201 **IGCSE 10-a-day: Model Solutions**

1 Relative speed

Oscar and Callum live 270 km apart.
At 10:00 they set off towards each other on the same road.
Oscar drives at 75 km/h and Callum drives at 60 km/h.
(a) At what time do they meet?
(b) How far from Oscar's home do they meet?

Closing speed = 75 + 60 = 135 km/h
(a) Time = 270 ÷ 135 = 2 hours = 120 minutes, so they meet at 12:00
(b) Distance = 75 × 2 = 150 km
Answer: (a) 12:00 (b) 150 km

2 Turning point of a quadratic by differentiation

$f(x) = -2x^2 + 8x + 12$
Use differentiation to find the coordinates of the turning point of the graph of $y = f(x)$. State whether it is a maximum or a minimum.

$f'(x) = -4x + 8$
$f'(x) = 0$: $-4x + 8 = 0$, so $x = 2$
$f(2) = 20$
$f''(x) = -4$, which is negative, so it is a maximum
Answer: (2, 20), maximum

3 Completing the square

(a) Write $x^2 + 12x + 38$ in the form $(x + p)^2 + q$
(b) Write down the coordinates of the turning point of the curve $y = x^2 + 12x + 38$

(a) Half of 12 is 6, so $x^2 + 12x = (x + 6)^2 - 36$
$x^2 + 12x + 38 = (x + 6)^2 - 36 + 38$
$= (x + 6)^2 + 2$
(b) The minimum is when $x + 6 = 0$, so $x = -6$ and $y = 2$
Answer: (a) $(x + 6)^2 + 2$ (b) $(-6, 2)$

4 Speed-time graphs

The graph shows the speed of a cyclist during 15 seconds.
(a) Work out the acceleration during the first 4 seconds.
(b) Work out the average speed over the whole 15 seconds.

(a) Acceleration = change in speed ÷ time = $(24 - 0) \div 4 = 6$ m/s^2
(b) Distance = area under the graph
Trapezium: $\frac{0 + 24}{2} \times 4 = 48$; rectangle: $24 \times 9 = 216$; triangle: $\frac{1}{2} \times 2 \times 24 = 24$
Total = 288 m, so average speed = 288 ÷ 15 = 19.2 m/s
Answer: (a) 6 m/s^2 (b) 19.2 m/s

5 Percentage change in area and volume

The radius of a circle is decreased by 50%.
Work out the percentage change in the area of the circle.

Radius multiplier = 0.5
Area multiplier = $0.5^2 = 0.25$
$0.25 - 1 = -0.75$, so a 75% decrease
Answer: 75% decrease

6 Solve f inverse equals g

$f(x) = \frac{x - 5}{4}$ and $g(x) = x^2 + 4x + 4$.

Solve $f^{-1}(x) = g(x)$.

$f^{-1}(x) = 4x + 5$

$4x + 5 = x^2 + 4x + 4$

$x^2 - 1 = 0$

$(x + 1)(x - 1) = 0$

$x = -1$ or $x = 1$

Answer: $x = -1$ or $x = 1$

7 Pythagoras with algebra

ABC is a right-angled triangle with sides $(x + 6)$ cm, $(x - 1)$ cm and $(x + 8)$ cm. The longest side is $(x + 8)$ cm.
(a) Find the value of x.
(b) Work out the area of the triangle.
The diagram is not drawn accurately.

(a) Pythagoras: $(x + 6)^2 + (x - 1)^2 = (x + 8)^2$

Expand and collect terms: $x^2 - 6x - 27 = 0$

Factorise: $(x - 9)(x + 3) = 0$

$x = 9$ or $x = -3$, and x must be positive, so $x = 9$

(b) The two shorter sides are 15 cm and 8 cm

Area $= \frac{1}{2} \times 15 \times 8 = 60$ cm^2

Answer: (a) $x = 9$ (b) 60 cm^2

8 Nature from the gradient either side

$y = x^3 - 3x^2 + 5$
(a) Show that there is a stationary point at $x = 2$.
(b) By finding the gradient when $x = \frac{3}{2}$ and when $x = \frac{5}{2}$, decide whether it is a maximum or a minimum.

$\frac{dy}{dx} = 3x^2 - 6x$, so when $x = 2$: $\frac{dy}{dx} = 0$

$x = \frac{3}{2}$: gradient $= -\frac{9}{4}$ (negative)

$x = \frac{5}{2}$: gradient $= \frac{15}{4}$ (positive)

The gradient goes from negative to 0 to positive, so (2, 1) is a minimum

Answer: (b) minimum

9 Transformed curve from a diagram

The diagram shows the graph of $y = f(x)$ with a minimum point at $(-1, 3)$.
The dashed curve C is the image of $y = f(x)$ after a translation.
The minimum point of C is $(-5, 0)$.
Write down the equation of C in terms of f.

The minimum point has moved from $(-1, 3)$ to $(-5, 0)$

Translation by $\begin{pmatrix} -4 \\ -3 \end{pmatrix}$

x moves left by 4: $f(x + 4)$. y moves down by 3: $- 3$

Answer: $y = f(x + 4) - 3$

10 Maximum speed from velocity

A particle moves in a straight line. Its velocity, v m/s, at time t seconds is given by

$v = 12t - 2t^2$ for $0 \leq t \leq 6$

Find the time at which the speed of the particle is greatest, and this greatest speed.

$a = \frac{dv}{dt} = -4t + 12$

Greatest speed when $a = 0$: $-4t + 12 = 0$, so $t = 3$

$\frac{d^2v}{dt^2} = -4 < 0$, so this is a maximum

$v = 12 \times 3 - 2 \times 3^2 = 18$

Answer: t = 3 s, greatest speed 18 m/s