Day 200 **IGCSE 10-a-day: Model Solutions**

1 f′(x) notation

f(x) = $3x^2 - 4x - 3$
(a) Find f′(x).
(b) Work out the value of f′(1).

(a) $3x^2 \to 6x$, $-4x \to -4$
f′(x) = $6x - 4$
(b) f′(1) = 2
Answer: (a) f′(x) = 6x − 4 (b) 2

2 Column vector arithmetic

$\mathbf{p} = \begin{pmatrix}6\\5\end{pmatrix}$, $\mathbf{q} = \begin{pmatrix}1\\-5\end{pmatrix}$ and $\mathbf{r} = \begin{pmatrix}2\\-1\end{pmatrix}$.
Work out $\mathbf{p} + 2\mathbf{q} - \mathbf{r}$.

$2\mathbf{q} = \begin{pmatrix}2\\-10\end{pmatrix}$

$\mathbf{p} + 2\mathbf{q} - \mathbf{r} = \begin{pmatrix}6\\5\end{pmatrix} + \begin{pmatrix}2\\-10\end{pmatrix} - \begin{pmatrix}2\\-1\end{pmatrix} = \begin{pmatrix}6\\-4\end{pmatrix}$

Answer: $\begin{pmatrix}\mathbf{6}\\\mathbf{-4}\end{pmatrix}$

3 Ordering numbers

Write these numbers in order of size, starting with the smallest.
$\frac{7}{8}$, 38%, 0.34, 0.64

Convert each to a decimal: $\frac{7}{8}$ = 0.875, 38% = 0.38, 0.34 = 0.34, 0.64 = 0.64

In order: 0.34, 38%, 0.64, $\frac{7}{8}$

Answer: 0.34, 38%, 0.64, $\frac{7}{8}$

4 Frequency density from a table

The table shows the masses of 120 parcels.
(a) Work out the frequency density for the class $70 < x \leq 80$.
(b) Estimate how many were greater than 55 kg.

(a) Frequency density = frequency ÷ class width = 10 ÷ 10 = 1
(b) In $50 < x \leq 70$, the part above 55 is 15 out of 20
Estimate = 1.7 × 15 = 25.5
Add the higher classes: 10, total = 35.5
Answer: (a) 1 (b) 35.5

5 Radius from chord and height of arc

A circular arc has a chord AB of length 32 cm.
The greatest distance between the chord and the arc is 4 cm.
Work out the radius of the circle.

Let the radius be r. The centre O is on the line through the midpoint M of AB and the highest point of the arc, so OM = r − 4
AM = 32 ÷ 2 = 16. Pythagoras in triangle OMA: $r^2 = (r - 4)^2 + 16^2$
$r^2 = r^2 - 8r + 16 + 256$
8r = 272, so r = 34 cm
Answer: 34 cm

6 Perpendicular bisector

A is the point (5, −5) and B is the point (13, 1).
Find an equation of the perpendicular bisector of AB.
Give your answer in the form ax + by = c, where a, b and c are integers.

Midpoint of AB = $(\frac{5+13}{2}, \frac{-5+1}{2}) = (9, -2)$

Gradient of AB = $\frac{1-(-5)}{13-5} = \frac{3}{4}$

The perpendicular gradient is $-1 \div \frac{3}{4} = -\frac{4}{3}$

Using the midpoint: $y + 2 = -\frac{4}{3}(x - 9)$

Multiply by 3 and rearrange: $4x + 3y = 30$

Answer: $4x + 3y = 30$

7 Curve touching the x-axis

The curve $y = 2x^3 - 18x^2 + 48x + k$, where k is a constant, has the x-axis as a tangent.
Find the two possible values of k.

$\frac{dy}{dx} = 6x^2 - 36x + 48 = 0$ gives x = 2 or x = 4

The x-axis is a tangent when a turning point has y = 0

x = 2: 40 + k = 0, so k = −40

x = 4: 32 + k = 0, so k = −32

Answer: k = −40 or k = −32

8 Exponential growth and decay

There are 20 insects in a sample. The number of insects is multiplied by 4 every day.
After how many days will there first be more than 40,000 insects?

Number after n days = 20×4^n

n = 5: 20,480

n = 6: 81,920

81,920 is the first value above 40,000

Answer: 6 days

9 Equation of a circle

A circle has centre C(4, −1) and passes through the point P(6, −5).
Find an equation of the circle.

$r^2 = CP^2 = (6 - 4)^2 + (-5 - (-1))^2$

$= 4 + 16 = 20$

$(x - 4)^2 + (y + 1)^2 = 20$

Answer: $(x - 4)^2 + (y + 1)^2 = 20$

10 Tangent parallel to a chord

The diagram shows part of the curve $y = 2x^2 - 4x + 2$. A and B are the points on the curve with x coordinates −1 and 1.
C is the point on the curve between A and B at which the tangent is parallel to AB. Find the coordinates of C.

A = (−1, 8) and B = (1, 0)

Gradient of AB = $\frac{0-8}{1-(-1)} = -4$

$\frac{dy}{dx} = 4x - 4 = -4$, so x = 0

y = 2

Answer: C (0, 2)