Day 20 **IGCSE 10-a-day: Model Solutions**

1 Solve f(x) = g(x)

f(x) = $3x^2 + 4x + 4$ and g(x) = $28x - 41$.
Solve f(x) = g(x).

$3x^2 + 4x + 4 = 28x - 41$
$3x^2 - 24x + 45 = 0$
$3(x - 3)(x - 5) = 0$
$x = 3$ or $x = 5$
Answer: x = 3 or x = 5

2 Terms of a quadratic sequence

The nth term of a sequence is $2n^2 + 8n + 6$.
(a) Work out the 11th term.
(b) Is 797 a term of the sequence? You must show your working.

(a) $2 \times 11^2 + 8 \times 11 + 6 = 336$
(b) Solve $2n^2 + 8n + 6 = 797$, so $2n^2 + 8n - 791 = 0$
$b^2 - 4ac = 8^2 - 4 \times 2 \times (-791) = 6392$, which is not a square number
So n is not a whole number, and 797 is not a term
Answer: (a) 336 (b) No

3 Exact lengths and areas

A square has sides of length $(7 + 2\sqrt{2})$ cm.
Work out the area of the square.
Give your answer in the form $a + b\sqrt{2}$

Area = $(7 + 2\sqrt{2})^2$
$= 49 + 28\sqrt{2} + 8$
$= 57 + 28\sqrt{2}$
Answer: $57 + 28\sqrt{2}$ cm^2

4 Chords and Pythagoras in circles

AB is a chord of a circle, centre O. M is the midpoint of AB. (Calculator)
(a) Work out the length OM.
(b) Work out angle AOB, to 1 decimal place.
The diagram is not drawn accurately.

(a) AM = 16 ÷ 2 = 8 cm
$OM^2 = OA^2 - AM^2 = 17^2 - 8^2 = 225$
OM = 15 cm

(b) sin(angle AOM) = $\frac{8}{17}$

Angle AOM = 28.07°, so angle AOB = 2 × 28.07 = 56.1°
Answer: (a) 15 cm (b) 56.1°

5 Minimum point of y = ax + b/x

The diagram shows a sketch of the curve $y = x + \frac{4}{x}$ for $x > 0$.

The curve has a minimum point at M.

Find the coordinates of M, justifying that it is a minimum.

$y = x + 4x^{-1}$, so $\frac{dy}{dx} = 1 - 4x^{-2}$

$\frac{dy}{dx} = 0$: $1 = \frac{4}{x^2}$, so $x^2 = 4$ and $x = 2$ ($x > 0$)

$\frac{d^2y}{dx^2} = 8x^{-3} = 1 > 0$, so a minimum

$y = 2 + \frac{4}{2} = 4$

Answer: M (2, 4)

6 Transformation by a matrix: image and description

$M = \begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix}$.

(a) Find the coordinates of the image of the point $(6, -1)$ under the transformation represented by M.

(b) Describe fully the single transformation represented by M.

(a) $\begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix}\begin{pmatrix} 6 \\ -1 \end{pmatrix} = \begin{pmatrix} -6 \\ -1 \end{pmatrix}$

The image is $(-6, -1)$

(b) The first column is where (1, 0) goes and the second column is where (0, 1) goes

M represents a reflection in the y-axis

Answer: (a) $(-6, -1)$ (b) Reflection in the y-axis

7 Using a line of best fit

The scatter graph shows age of car (x) and value (y).

The equation of the line of best fit is $y = -1.5x + 26$.

(a) Estimate y when $x = 5$.

(b) Explain why an estimate of y when $x = 25$ may be unreliable.

(a) $y = -1.5 \times 5 + 26 = 18.5$

(b) $x = 25$ is outside the range of the data on the graph, so the estimate relies on the pattern continuing (extrapolation)

Answer: (a) 18.5 (b) x = 25 is outside the range of the data

8 Minimum surface area of a tank

A tank is a cuboid with a square base of side x metres and height h metres, and has an open top. Its volume is 32 m^3.

(a) Show that the total outside surface area, A m^2, is $A = x^2 + \frac{128}{x}$.

(b) Find the minimum possible value of A, showing that it is a minimum.

$V = x^2h = 32$, so $h = \frac{32}{x^2}$

$A = x^2 + 4xh = x^2 + 4x \times \frac{32}{x^2} = x^2 + \frac{128}{x}$

$\frac{dA}{dx} = 2x - 128x^{-2} = 0$ gives $x^3 = 64$, so $x = 4$

$\frac{d^2A}{dx^2} = 2 + 256x^{-3} = 6 > 0$, so a minimum

$A = 48$

Answer: 48 m^2

9 Gradient of a chord

A and B are points on the curve $y = 3x^2 + 4x$.
The x coordinate of A is 2 and the x coordinate of B is 5.
Work out the gradient of the line AB.

At A: $y = 20$. At B: $y = 95$
A is (2, 20) and B is (5, 95)
Gradient = $\frac{95 - 20}{5 - 2} = \frac{75}{3} = 25$

Answer: 25

10 Tangent meets the curve again (cubic)

The tangent to the curve $y = x^3 - x^2 + 6x + 4$ at the point P where $x = -1$ meets the curve again at Q.
Find the coordinates of Q.

$\frac{dy}{dx} = 3x^2 - 2x + 6$, so at P the gradient is 11 and $y = -4$
Tangent: $y = 11x + 7$
Intersection: $x^3 - x^2 - 5x - 3 = 0$
$x = -1$ is a repeated root, so the roots are -1, -1 and x_Q
Sum of roots = 1, so $-2 + x_Q = 1$ and $x_Q = 3$
$y = 40$
Answer: (3, 40)