**Day 2** **IGCSE 10-a-day: Model Solutions**

## 1 Negative powers

Differentiate $-6x^3 - \frac{8}{x^2}$ with respect to x.

Write negative powers as indices: $y = -6x^3 - 8x^{-2}$

$-6x^3 \rightarrow -18x^2$, $-8x^{-2} \rightarrow 16x^{-3}$

$\frac{dy}{dx} = -18x^2 + 16x^{-3}$

**Answer:** $\mathbf{\frac{dy}{dx} = -18x^2 + \frac{16}{x^3}}$

## 2 Function values and composites

f(x) = 5x + 6 and g(x) = $2x^2 + 5$.
Work out ff(−2) + gf(2).

f(−2) = −4, so ff(−2) = f(−4) = −14
f(2) = 16, so gf(2) = g(16) = 517
−14 + 517 = 503
**Answer: 503**

## 3 Fencing with a partition

A rectangular enclosure is x metres wide and y metres long. It is fenced all round and divided into three equal parts by two fences each of length x metres, parallel to the width.
The total length of fencing used is 16 m.
(a) Show that the area of the enclosure is A = $-2x^2 + 8x$.
(b) Find the maximum possible area, justifying that it is a maximum.

Fencing: $4x + 2y = 16$, so $y = \frac{16 - 4x}{2}$

$A = xy = \frac{x(16 - 4x)}{2} = -2x^2 + 8x$

$\frac{dA}{dx} = -4x + 8 = 0$ gives x = 2

$\frac{d^2A}{dx^2} = -4 < 0$, so a maximum

A = 8
**Answer: 8 m²**

## 4 Averages with algebra

Here are five numbers: x − 1, 4, 3, 2x + 6, 3x + 6.
The mean of the five numbers is 12.
(a) Work out the value of x.
(b) Work out the range of the five numbers.

(a) Total = (x − 1) + (4) + (3) + (2x + 6) + (3x + 6) = 12 × 5 = 60
6x + 18 = 60, so 6x = 42 and x = 7
The numbers are 6, 4, 3, 20, 27
(b) Range = 27 − 3 = 24
**Answer: (a) x = 7 (b) 24**

## 5 Circle theorems: angle at the centre

A, B and C are points on a circle, centre O. Angle ACB = 39°.
(a) Work out the size of angle AOB.
(b) Work out the size of angle OAB.
The diagram is not drawn accurately.

(a) The angle at the centre is twice the angle at the circumference: 2 × 39 = 78°
(b) OA = OB (radii), so triangle OAB is isosceles
Angle OAB = (180 − 78) ÷ 2 = 51°
**Answer: (a) 78° (b) 51°**

## 6 Intersection and gradient

The curve $y = 2x^2 + 4x + 5$ and the line $y = 10x + 5$ intersect at two points A and B, where A has the smaller x coordinate.
(a) Find the coordinates of A and B.
(b) Find the gradient of the curve at A and at B.

$2x^2 + 4x + 5 = 10x + 5$ gives $2x^2 - 6x = 0$, so $2x(x - 3) = 0$
$x = 0$ or $x = 3$, giving A (0, 5) and B (3, 35)
$\frac{dy}{dx} = 4x + 4$: at A it is 4, at B it is 16
**Answer: (a) A (0, 5), B (3, 35) (b) 4 at A, 16 at B**

## 7 3D trigonometry: cuboid

ABCDEFGH is a cuboid. AB = 11 cm, BC = 13 cm and CG = 5 cm.
(a) Work out the length of AC.
(b) Work out the size of the angle between AG and the base ABCD.
Give answers to 3 significant figures or 1 decimal place. (Calculator)

(a) $AC^2 = 11^2 + 13^2 = 290$, so AC = 17.03 cm
(b) CG is perpendicular to the base, so the angle is angle GAC in right-angled triangle ACG
$\tan GAC = \frac{CG}{AC} = \frac{5}{17.03}$
Angle = 16.4°
**Answer: (a) 17.0 cm (b) 16.4°**

## 8 Perpendicular bisector

A is the point (−4, 0) and B is the point (−8, 6).
Find an equation of the perpendicular bisector of AB.
Give your answer in the form ax + by = c, where a, b and c are integers.

Midpoint of AB = $(\frac{-4 + (-8)}{2}, \frac{0 + 6}{2}) = (-6, 3)$
Gradient of AB = $\frac{6 - 0}{-8 - (-4)} = -\frac{3}{2}$
The perpendicular gradient is $-1 \div (-\frac{3}{2}) = \frac{2}{3}$
Using the midpoint: $y - 3 = \frac{2}{3}(x + 6)$
Multiply by 3 and rearrange: $-2x + 3y = 21$
Multiply by −1: $2x - 3y = -21$
**Answer: $2x - 3y = -21$**

## 9 Combining means

The mean of the marks in a test for the men is 77.
The mean of the marks in a test for the women is 73.
The mean for all of the men and women together is 74.2.
Work out the ratio of the number of men to the number of women.

Let there be p men and q women
Total = 77p + 73q, and the overall mean gives $77p + 73q = 74.2(p + q)$
$2.8p = 1.2q$
$p : q = 1.2 : 2.8 = 3 : 7$
**Answer: 3 : 7**

## 10 Surds and fractions

Simplify $\frac{1}{4+\sqrt{10}} + \frac{1}{4-\sqrt{10}}$

Give your answer with a rational denominator.

Common denominator: $(4+\sqrt{10})(4-\sqrt{10}) = 16 - 10 = 6$

Numerator: $(4-\sqrt{10}) + (4+\sqrt{10})$

$= 8$

$\frac{8}{6} = \frac{4}{3}$

**Answer:** $\mathbf{\frac{4}{3}}$