Day 199 **IGCSE 10-a-day: Model Solutions**

1 Vectors in a triangle with a diagram

In triangle OAB, $\mathbf{OA} = \mathbf{a}$ and $\mathbf{OB} = \mathbf{b}$.
M is on OA so that OM : MA = 2 : 1.
N is on OB so that ON : NB = 2 : 1.
Find $\mathbf{MN}$ in terms of $\mathbf{a}$ and $\mathbf{b}$.

$\mathbf{OM} = \frac{2}{3}\mathbf{a}$ and $\mathbf{ON} = \frac{2}{3}\mathbf{b}$

$\mathbf{MN} = \mathbf{MO} + \mathbf{ON} = -\mathbf{OM} + \mathbf{ON}$

$\mathbf{MN} = -\frac{2}{3}\mathbf{a} + \frac{2}{3}\mathbf{b}$

Answer: $\mathbf{-\frac{2}{3}a + \frac{2}{3}b}$

2 Rate of change of area or volume

The volume, V cm^3, of a cube of edge x cm is $V = x^3$.

(a) Find $\frac{dV}{dx}$ when x = 5.

(b) Find the value of x when $\frac{dV}{dx} = 12$.

$\frac{dV}{dx} = 3x^2$

When x = 5: $\frac{dV}{dx} = 75$

$3x^2 = 12$, so $3x^2 = 12$ and x = 2 (lengths are positive)

Answer: (a) 75 (b) x = 2

3 Angles in polygons

The interior angles of a pentagon are
x°, (x + 40)°, (x − 20)°, (x + 20)°, (2x − 40)°
(a) Work out the value of x.
(b) Work out the size of the largest angle.

(a) Sum of interior angles = (5 − 2) × 180 = 540°
6x = 540, so 6x = 540
x = 90
(b) The angles are 90°, 130°, 70°, 110°, 140°
The largest is 140°

Answer: (a) x = 90 (b) 140°

4 Standard form

Work out $(2 \times 10^2) + (2.1 \times 10^1)$
Give your answer in standard form.

Write the first number as a multiple of 10^1: $2 \times 10^2 = 20 \times 10^1$
$(20 + 2.1) \times 10^1 = 22.1 \times 10^1$
$= 2.21 \times 10^2$

Answer: $\mathbf{2.21 \times 10^2}$

5 Simplifying algebraic fractions

Simplify fully $\frac{5x^2 - 125}{x^2 - 13x + 40}$

Factorise the top: 5(x − 5)(x + 5)
Factorise the bottom: (x − 5)(x − 8)
Cancel the common factor (x − 5): $\frac{5(x + 5)}{x - 8}$

Answer: $\mathbf{\frac{5(x + 5)}{x - 8}}$

6 Find unknown coefficients from gradients

The curve $y = x^3 + ax^2 + bx$ has gradient 6 when $x = 0$ and gradient 9 when $x = 3$.
Find the values of a and b.

$\frac{dy}{dx} = 3x^2 + 2ax + b$

$x = 0$: $b = 6$

$x = 3$: $6a + b = -18$

Subtract and solve: $a = -4$

$b = 6$

Answer: $a = -4$, $b = 6$

7 Solving AX = B

$A = \begin{pmatrix} 1 & -3 \\ 3 & 0 \end{pmatrix}$ and $B = \begin{pmatrix} 1 \\ 12 \end{pmatrix}$.

AX = B, where X is a 2 × 1 matrix.
Find X.

$X = A^{-1}B$ (multiply both sides on the left by A^{-1})

$A^{-1} = \frac{1}{9}\begin{pmatrix} 0 & 3 \\ -3 & 1 \end{pmatrix}$

$X = \frac{1}{9}\begin{pmatrix} 0 & 3 \\ -3 & 1 \end{pmatrix}\begin{pmatrix} 1 \\ 12 \end{pmatrix} = \begin{pmatrix} 4 \\ 1 \end{pmatrix}$

Answer: $X = \begin{pmatrix} \mathbf{4} \\ \mathbf{1} \end{pmatrix}$

8 Product of brackets: tangent gradient

$y = x(x + 2)(2x - 1)$
Work out the gradient of the curve at the point where $x = 1$.

Expand: $y = 2x^3 + 3x^2 - 2x$

$\frac{dy}{dx} = 6x^2 + 6x - 2$

$x = 1$: $\frac{dy}{dx} = 10$

Answer: 10

9 Mixtures and percentage profit

A shop mixes two types of coffee.
Type A costs £6.00 per kg and type B costs £9.00 per kg.
They are mixed in the ratio A : B = 1 : 2.
The mixture is sold for £14.40 per kg.
Work out the percentage profit.

Cost of 3 kg of mixture = 1 × £6.00 + 2 × £9.00 = £24.00

Cost per kg = £24.00 ÷ 3 = £8.00

Profit per kg = £14.40 − £8.00 = £6.40

Percentage profit = 6.40 ÷ 8.00 × 100 = 80%

Answer: 80%

10 Cone made from a sector

A sector of a circle has radius 30 cm and angle 72°.
The sector is made into a cone by joining the two straight edges together.
(a) Work out the radius of the base of the cone.
(b) Work out the exact volume of the cone. Give your answer in terms of π, as a surd if necessary.

(a) Arc length of the sector = $\frac{72}{360} \times 2\pi \times 30 = 12\pi$ cm

This is the circumference of the base: $2\pi r = 12\pi$, so r = 6 cm

(b) Slant height = 30 cm, so height$^2 = 30^2 - 6^2 = 864$, height = $12\sqrt{6}$ cm

Volume = $\frac{1}{3} \times \pi \times 6^2 \times 12\sqrt{6} = 144\sqrt{6}\pi$ cm^3

Answer: (a) 6 cm (b) $144\sqrt{6}\pi$ cm^3