Day 198 **IGCSE 10-a-day: Model Solutions**

1 Recurring decimals

Show that $0.3\dot{6} = \frac{11}{30}$

Let x = 0.3666...
10x = 3.6666...
100x = 36.6666...
Subtract: 90x = 33
$x = \frac{33}{90} = \frac{11}{30}$

Answer: x = $\frac{11}{30}$

2 Area of a segment

The circle has centre O and radius 8 cm. A and B are points on the circle and angle AOB = 100°. (Calculator)
Work out the area of the shaded segment. Give your answer correct to 3 significant figures.
The diagram is not drawn accurately.

Area of sector OAB = $\frac{100}{360} \times \pi \times 8^2 = 55.851$ cm^2

Area of triangle OAB = $\frac{1}{2} \times 8 \times 8 \times \sin 100° = 31.514$ cm^2

Segment = sector − triangle = 55.851 − 31.514 = 24.337
= 24.3 cm^2 (3 s.f.)

Answer: 24.3 cm^2

3 Integers satisfying a double inequality

n is an integer.
How many values of n satisfy
$-10 \leq n - 5 \leq -3$

Add 5 from each part: $-5 \leq n \leq 2$
Divide each part by 1: $-5 \leq n \leq 2$
Integers: −5, −4, −3, −2, −1, 0, 1, 2
Count: 8

Answer: 8

4 Open box from a square sheet

Squares of side x cm are cut from the corners of a 30 cm square card sheet, which is folded into an open box of volume V cm^3.

(a) Show that $V = 4x^3 - 120x^2 + 900x$.
(b) Find x for the maximum volume, and this volume.

Base is a square of side 30 − 2x, so $V = x(30 - 2x)^2 = x(900 - 120x + 4x^2) = 4x^3 - 120x^2 + 900x$

$\frac{dV}{dx} = 12x^2 - 240x + 900 = 0$, so $x^2 - 20x + 75 = 0$

$12(x - 5)(x - 15) = 0$, so x = 5 or x = 15

x = 15 is rejected (15 is more than half of 30). $\frac{d^2V}{dx^2} = 24x - 240$, which is −120 when x = 5, so a maximum

V = 2000

Answer: x = 5, V = 2000 cm^3

5 Completing the square

(a) Write $2x^2 - 12x + 13$ in the form $a(x + p)^2 + q$

(b) Write down the coordinates of the turning point of the curve $y = 2x^2 - 12x + 13$

(a) Take out the factor 2 from the x terms: $2(x^2 - 6x) + 13$

$x^2 - 6x = (x - 3)^2 - 9$, so $2x^2 - 12x + 13 = 2[(x - 3)^2 - 9] + 13$

$= 2(x - 3)^2 - 5$

(b) The minimum is when $x - 3 = 0$, so $x = 3$ and $y = -5$

Answer: (a) $2(x - 3)^2 - 5$ (b) $(3, -5)$

6 Expanding brackets with surds

Show that $(5 - 3\sqrt{2})(5 + 3\sqrt{2})$ is an integer.

Use the difference of two squares: $(5 - 3\sqrt{2})(5 + 3\sqrt{2}) = 25 - (3\sqrt{2})^2$

$= 25 - 18 = 7$

Answer: 7

7 Increasing range for y = ax + b/x

$y = 4x + \frac{16}{x}$

Find the values of x for which y is increasing.

$\frac{dy}{dx} = 4 - \frac{16}{x^2}$

$\frac{dy}{dx} > 0$: $4 > \frac{16}{x^2}$

$x^2 > 4$ (x^2 is positive, so multiply through)

$x < -2$ or $x > 2$

Answer: $x < -2$ or $x > 2$

8 Solve f inverse equals g

$f(x) = \frac{x - 5}{3}$ and $g(x) = x^2 + 6x + 7$.

Solve $f^{-1}(x) = g(x)$.

$f^{-1}(x) = 3x + 5$

$3x + 5 = x^2 + 6x + 7$

$x^2 + 3x + 2 = 0$

$(x + 2)(x + 1) = 0$

$x = -2$ or $x = -1$

Answer: $x = -2$ or $x = -1$

9 Increasing for all x

$f(x) = x^3 + kx^2 + 75x$, where k is a constant.

Given that f is an increasing function for all values of x, find the range of possible values of k.

$f'(x) = 3x^2 + 2kx + 75$

f is increasing for all x when $f'(x) > 0$ for all x, so the quadratic has no real roots

Discriminant: $(2k)^2 - 4 \times 3 \times 75 < 0$, so $4k^2 < 900$

$k^2 < 225$, so $-15 < k < 15$

Answer: $-15 < k < 15$

10 Estimated mean with an unknown frequency

The table gives information about the times taken by some people to travel to work.
An estimate for the mean is 10.
Work out the value of x.

Mid-points: 2.5, 7.5, 12.5, 17.5
Estimated mean = $\Sigma fm \div \Sigma f$, so $(362.5 + 12.5x) \div (37 + x) = 10$
$362.5 + 12.5x = 10(37 + x) = 370 + 10x$
$(12.5 - 10)x = 370 - 362.5$
$2.5x = 7.5$, so $x = 3$
Answer: x = 3