Day 197 **IGCSE 10-a-day: Model Solutions**

1 HCF and LCM

(a) Write 140 and 245 as products of prime factors.
(b) Find the HCF and the LCM of 140 and 245.
Give your answers as numbers.

(a) $140 = 2^2 \times 5 \times 7$ and $245 = 5 \times 7^2$
(b) HCF $= 5 \times 7 = 35$
LCM $= 2^2 \times 5 \times 7^2 = 980$
Answer: (a) $2^2 \times 5 \times 7$; 5×7^2 (b) HCF = 35, LCM = 980

2 Exact trigonometric values

Work out the exact value of $6 \tan 45° \div \sin 30°$.
Give your answer in its simplest form, with no decimals.

$\tan 45° = 1$ and $\sin 30° = \frac{1}{2}$
Substitute and simplify (rationalise any denominator)
$= 12$
Answer: 12

3 Angles with algebra

In triangle PQR, side QR is extended to S.
Angle QPR = $(5x)°$
Angle PQR = $(3x - 15)°$
Angle PRS = $(6x + 19)°$
(a) Work out the value of x.
(b) Work out the size of angle PRQ.

(a) The exterior angle of a triangle equals the sum of the two opposite interior angles
$6x + 19 = 8x - 15$
$2x = 34$
$x = 17$
(b) Angle PRS = 121° so angle PRQ = $180° - 121° = 59°$
Answer: (a) x = 17 (b) 59°

4 Stationary points of a cubic

A curve has equation $y = 2x^3 + 3x^2 - 72x - 3$.
Find the coordinates of the stationary points of the curve.

$\frac{dy}{dx} = 6x^2 + 6x - 72$

Stationary points: $\frac{dy}{dx} = 0$, so $6(x + 4)(x - 3) = 0$

$x = -4$ or $x = 3$
$y = 205$ when $x = -4$ and $y = -138$ when $x = 3$
Answer: (−4, 205) and (3, −138)

5 Transformation by a matrix: image and description

$M = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$.

(a) Find the coordinates of the image of the point (−2, −4) under the transformation represented by M.
(b) Describe fully the single transformation represented by M.

(a) $\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}\begin{pmatrix} -2 \\ -4 \end{pmatrix} = \begin{pmatrix} -4 \\ -2 \end{pmatrix}$
The image is (−4, −2)
(b) The first column is where (1, 0) goes and the second column is where (0, 1) goes
M represents a reflection in the line y = x
Answer: (a) (−4, −2) (b) Reflection in the line y = x

6 Algebraic fractions: multiplying and dividing

Simplify fully $\frac{x^2 - 3x - 10}{x^2 - 7x} \times \frac{x^2 + 4x}{x - 5}$

Factorise: $\frac{(x - 5)(x + 2)}{x(x - 7)} \times \frac{x(x + 4)}{(x - 5)}$

Cancel $(x - 5)$ and x

$= \frac{(x + 2)(x + 4)}{(x - 7)}$

Answer: $\mathbf{\frac{(x + 2)(x + 4)}{(x - 7)}}$

7 Open box of fixed surface area

An open box has a square base of side x cm and height h cm. The total outside surface area is 432 cm^2.

(a) Show that the volume of the box is $V = \frac{1}{4}(432x - x^3)$ cm^3.

(b) Find the maximum volume of the box, proving that it is a maximum.

$x^2 + 4xh = 432$, so $h = \frac{432 - x^2}{4x}$

$V = x^2h = \frac{x(432 - x^2)}{4} = \frac{1}{4}(432x - x^3)$

$\frac{dV}{dx} = -\frac{3}{4}x^2 + 108 = 0$, so $x^2 = 144$ and $x = 12$

$\frac{d^2V}{dx^2} = -\frac{3}{2}x = -18 < 0$, so a maximum

$V = 864$

Answer: 864 cm^3

8 Tangent to a circle

A circle has equation $x^2 + y^2 = 289$.
The point P(8, −15) lies on the circle.
(a) Find the equation of the tangent to the circle at P. Give your answer in the form $ax + by = c$, where a, b and c are integers.
(b) The tangent meets the x-axis at the point T. Write down the coordinates of T.

(a) The gradient of the radius OP is $-\frac{15}{8}$

The tangent is perpendicular to the radius, so its gradient is $-1 \div (-\frac{15}{8}) = \frac{8}{15}$

$y - (-15) = \frac{8}{15}(x - 8)$

Multiply out and rearrange: $8x - 15y = 289$

(b) On the x-axis $y = 0$, so $8x = 289$

$x = \frac{289}{8}$, so T is $(\frac{289}{8}, 0)$

Answer: (a) $8x - 15y = 289$ (b) T = $(\frac{289}{8}, 0)$

9 Curves that touch

The curves $y = x^2 + 3x + 4$ and $y = -2x^2 + 15x - 8$ touch at one point.
Find the coordinates of this point and the equation of the common tangent.

Equate the two expressions: $3x^2 - 12x + 12 = 0$

That is $3(x - 2)^2 = 0$ with a repeated root $x = 2$

$y = 14$, so the point is (2, 14)

$\frac{dy}{dx} = 2x + 3$ on the first curve: gradient 7

Tangent: $y = 7x$

Answer: (2, 14), $y = 7x$

10 Reverse percentage growth and decay

A laptop loses 20% of its value each year.
After 3 years it is worth £4,608.
Work out its value when it was new. (Calculator)

Yearly multiplier = 0.8

Original × 0.8^3 = £4,608

Original = £4,608 ÷ 0.8^3 = £9,000

Answer: £9,000