Day 196 **IGCSE 10-a-day: Model Solutions**

1 Equation of a tangent

A curve has equation $y = x^2 - 4x - 3$.
Find the equation of the tangent to the curve at the point where $x = 0$.

When $x = 0$, $y = -3$
$\frac{dy}{dx} = 2x - 4$, so the gradient at $x = 0$ is -4
$y + 3 = -4x$
$y = -4x - 3$
Answer: $y = -4x - 3$

2 Distance-time graphs

The graph shows Tariq's cycle ride to a café and back.
(a) Work out Tariq's speed on the way back.
(b) Work out Tariq's average speed for the whole ride, including the stop.

(a) Return: 15 km in 1.5 hours, so speed = 15 ÷ 1.5 = 10 km/h
(b) Total distance = 2 × 15 = 30 km, total time = 3 hours
Average speed = 30 ÷ 3 = 10 km/h
Answer: (a) 10 km/h (b) 10 km/h

3 Divide then differentiate

$y = \frac{x^4 - 2x + 3}{x^3}$

Find $\frac{dy}{dx}$.

Divide each term by x^3: $y = x - 2x^{-2} + 3x^{-3}$
$x \to 1$, $-2x^{-2} \to 4x^{-3}$, $3x^{-3} \to -9x^{-4}$
$\frac{dy}{dx} = 1 + 4x^{-3} - 9x^{-4}$
Answer: $\frac{dy}{dx} = 1 + \frac{4}{x^3} - \frac{9}{x^4}$

4 Speed-time graphs

The graph shows the speed of a train between two stations.
(a) Work out the acceleration in the first 5 seconds.
(b) Work out the total distance.

(a) Acceleration = change in speed ÷ time = 15 ÷ 5 = 3 m/s^2
(b) Distance = area under the graph, split into two triangles and a rectangle
$\frac{1}{2} \times 5 \times 15 + 15 \times 6 + \frac{1}{2} \times 3 \times 15 = \frac{75}{2} + 90 + \frac{45}{2}$
$= 150$
Answer: (a) 3 m/s^2 (b) 150 m

5 Three-part ratios

Dina, Leila and Sofia share some sweets.
Dina : Leila = 4 : 9
Leila : Sofia = 7 : 6
(a) Work out the ratio Dina : Leila : Sofia.
(b) Leila gets 105 sweets more than Dina. How many sweets does Sofia get?

(a) Make the Leila part the same: 4 : 9 = 28 : 63 and 7 : 6 = 63 : 54
Dina : Leila : Sofia = 28 : 63 : 54
(b) Difference in parts = 35, so 1 part = 105 ÷ 35 = 3
Sofia gets 54 × 3 = 162 sweets
Answer: (a) 28 : 63 : 54 (b) 162 sweets

6 Index equations

Solve $2^{x+2} = 16^0$

Write both sides as powers of 2: $16 = 2^4$
Equate the powers: $x + 2 = 0$
$x = -2$
Answer: $x = -2$

7 Triangle shapes from position vectors

A = (−1, 1), B = (−6, −4) and C = (4, −4).
Show that triangle ABC is isosceles and right-angled.

$\mathbf{AB} = \begin{pmatrix} -5 \\ -5 \end{pmatrix}$ and $\mathbf{AC} = \begin{pmatrix} 5 \\ -5 \end{pmatrix}$

$|AB|^2 = 25 + 25 = 50$ and $|AC|^2 = 25 + 25 = 50$
$|AB| = |AC| = \sqrt{50}$, so the triangle is isosceles
$\mathbf{BC} = \begin{pmatrix} 10 \\ 0 \end{pmatrix}$ so $|BC|^2 = 100 + 0 = 100$
$|AB|^2 + |AC|^2 = 50 + 50 = 100 = |BC|^2$, so the angle at A is 90° (Pythagoras)
Answer: $|\mathbf{AB}| = |\mathbf{AC}|$ and $\mathbf{AB}^2 + \mathbf{AC}^2 = \mathbf{BC}^2$

8 Curve touching the x-axis

The curve $y = -2x^3 + 6x^2 + k$, where k is a constant, has the x-axis as a tangent.
Find the two possible values of k.

$\frac{dy}{dx} = -6x^2 + 12x = 0$ gives $x = 0$ or $x = 2$
The x-axis is a tangent when a turning point has $y = 0$
$x = 0$: $0 + k = 0$, so $k = 0$
$x = 2$: $8 + k = 0$, so $k = -8$
Answer: $k = 0$ or $k = -8$

9 Largest domain and range

$f(x) = \sqrt{x + 5} - 2$.
(a) State the largest possible domain of f.
(b) State the range of f for this domain.

(a) The expression under the square root must be at least 0: $x + 5 \geq 0$
Domain: $x \geq -5$
(b) A square root is never negative, so the least value of f is −2
Answer: (a) $x \geq -5$ (b) $f(x) \geq -2$

10 Areas of similar triangles

In the diagram, DE is parallel to BC. AD = 4 cm, DB = 10 cm and the area of triangle ADE is 36 cm^2.
Work out the area of the trapezium DBCE.
The diagram is not drawn accurately.

Triangles ADE and ABC are similar. AB = 4 + 10 = 14 cm

Length scale factor = $14 \div 4 = \frac{7}{2}$

Area scale factor = $(\frac{7}{2})^2 = \frac{49}{4}$

Area of triangle ABC = $36 \times \frac{49}{4} = 441$ cm^2

Trapezium = $441 - 36 = 405$ cm^2
Answer: 405 cm^2