Day 195 **IGCSE 10-a-day: Model Solutions**

1 Nature from the gradient either side

$y = 2x^3 + 3x^2 - 6$

(a) Show that there is a stationary point at $x = -1$.

(b) By finding the gradient when $x = -\frac{3}{2}$ and when $x = -\frac{1}{2}$, decide whether it is a maximum or a minimum.

$\frac{dy}{dx} = 6x^2 + 6x$, so when $x = -1$: $\frac{dy}{dx} = 0$

$x = -\frac{3}{2}$: gradient $= \frac{9}{2}$ (positive)

$x = -\frac{1}{2}$: gradient $= -\frac{3}{2}$ (negative)

The gradient goes from positive to 0 to negative, so $(-1, -5)$ is a maximum

Answer: (b) maximum

2 Equations with fractions

Solve $\frac{x-3}{3} + \frac{x-5}{2} = -1$

Multiply every term by 6: $2(x - 3) + 3(x - 5) = -6$

Expand: $5x - 21 = -6$

$5x = 15$

$x = 3$

Answer: x = 3

3 Rate of change of area or volume

A cylinder has radius r cm and fixed height 6 cm. Its volume is $V = 6\pi r^2$ cm^3.

(a) Find $\frac{dV}{dr}$ when $r = 9$.

(b) Find the value of r when $\frac{dV}{dr} = 108\pi$.

$\frac{dV}{dr} = 12\pi r$

When $r = 9$: $\frac{dV}{dr} = 108\pi$

$12\pi r = 108\pi$, so $12r = 108$ and $r = 9$ (lengths are positive)

Answer: (a) 108π (b) r = 9

4 Variation: missing values

y is directly proportional to x.

When $x = 2$, $y = 12$.

When $x = 3$, $y = a$.

When $x = b$, $y = 36$.

Work out the values of a and b.

$y = kx$, $k = 12 \div 2 = 6$

$a = 6 \times 3 = 18$

$36 = 6b$ so $b = 6$, $b = 6$

Answer: a = 18, b = 6

5 Stationary points of y = ax + b/x

$y = -3x - \frac{12}{x}$

Find the coordinates of the stationary points and determine whether each is a maximum or a minimum.

$\frac{dy}{dx} = -3 + \frac{12}{x^2}$

$\frac{dy}{dx} = 0$: $\frac{12}{x^2} = 3$, so $x^2 = 4$ and $x = 2$ or $x = -2$

$y = -12$ when $x = 2$ and $y = 12$ when $x = -2$

$\frac{d^2y}{dx^2} = -\frac{24}{x^3}$

$x = 2$: $\frac{d^2y}{dx^2} = -3$ so maximum; $x = -2$: $\frac{d^2y}{dx^2} = 3$ so minimum

Answer: (2, −12) maximum; (−2, 12) minimum

6 Reverse percentages

The value of a house is decreased by 5% in one year.
The next year its value is increased by 30%.
The next year its value is increased by 40%.
It is now worth £466,830.
Work out the value of the house before the first change.

Multipliers: 0.95, 1.3, 1.4
Overall multiplier = 0.95 × 1.3 × 1.4 = 1.729
Original = final ÷ multiplier = £466,830 ÷ 1.729
= £270,000

Answer: £270,000

7 Shaded areas: exact answers

The diagram shows a square inside a circle of radius 7 cm.
The four vertices of the square lie on the circle.
Work out the area of the shaded region. Give your answer in terms of π.

The diagonal of the square is a diameter of the circle: 14 cm

Area of the square = $\frac{1}{2}$ × diagonal × diagonal = $\frac{1}{2}$ × 14 × 14 = 98

Area of circle = $\pi \times 7^2 = 49\pi$

Shaded area = $49\pi - 98$ cm^2

Answer: $(49\pi - 98)$ cm^2

8 nth term of a cubic sequence

Here are the first five terms of a sequence.
4, 13, 46, 115, 232
The nth term is of the form $an^3 + bn + c$.
Find the values of a, b and c.

1st differences: 9, 33, 69, 117. 2nd: 24, 36, 48. 3rd: 12, 12
Third differences = 6a = 12 so a = 2
Subtract $2n^3$: 2, −3, −8, −13, −18
This is linear with common difference −5, so b = −5, and c = 2 − (−5) = 7

Answer: a = 2, b = −5, c = 7

9 Venn diagrams with algebra

The Venn diagram shows how many of 35 students have a tablet (T) or a laptop (L).

(a) Work out the value of x.

(b) A student is chosen at random from those who have a laptop. Work out the probability that this student also has a tablet.

(a) Total: $(x + 1) + x + (2x - 1) + 7 = 35$

$4x + 7 = 35$, so $x = 7$

(b) Students in the second circle: $x + 2x - 1 = 7 + 13 = 20$, of whom 7 are in both

Probability = $\frac{7}{20}$

Answer: (a) x = 7 (b) $\frac{7}{20}$

10 Vector proof: collinear points

OA = $\mathbf{a}$ and OB = $\mathbf{b}$.

OP = $2\mathbf{a} - 2\mathbf{b}$, OQ = $-4\mathbf{a} + \mathbf{b}$ and OR = $-6\mathbf{a} + 2\mathbf{b}$.

(a) Show that P, Q and R lie on a straight line.

(b) Work out the ratio PQ : QR in its simplest form.

(a) PQ = OQ − OP = $-6\mathbf{a} + 3\mathbf{b}$

QR = OR − OQ = $-2\mathbf{a} + \mathbf{b}$

QR = $\frac{1}{3}$ PQ, so PQ and QR are parallel, and they share the point Q

So P, Q and R lie on a straight line

(b) PQ : QR = 3 : 1

Answer: (b) 3 : 1