Day 194 **IGCSE 10-a-day: Model Solutions**

1 Time zones

A flight leaves London at 19:15 (London time).
It takes 6 hours 55 minutes.
When it is 12:00 in London it is 07:00 in New York.
At what time, in New York time, does the flight land? Say if it is on a different day.

Arrival in London time = 19:15 + 6 h 55 min = 02:10 the next day
New York is 5 hours behind London, so subtract 5 hours
21:10
Answer: 21:10 the same day

2 Equations with fractions

Solve $\frac{x+1}{3} + \frac{x+4}{6} = 2$

Multiply every term by 6: $2(x + 1) + (x + 4) = 12$
Expand: $3x + 6 = 12$
$3x = 6$
$x = 2$
Answer: $x = 2$

3 Writing inequalities from a situation

On a ferry there are x adults and y children.
There are at most 40 people. There are at least 3 children for every adult. There are at least 4 adults.
(a) Write down three inequalities for x and y.
(b) Find the greatest number of children on the ferry.

(a) At most 40 people: $x + y \leq 40$
At least 3 children for every adult: $y \geq 3x$
At least 4 adults: $x \geq 4$
(b) The most children occurs with the fewest adults: $x = 4$
$y \leq 40 - 4 = 36$ and $y \geq 12$, so $y = 36$ is allowed
Answer: (a) $x + y \leq 40$, $y \geq 3x$, $x \geq 4$ (b) 36

4 Tangent parallel to a given line

The tangents at two points A and B on the curve $y = 2x^3 + 15x^2 + 34x + 7$ are parallel to the line $y = -2x - 7$.
Find the coordinates of A and B.

Gradient of the line is -2, so $\frac{dy}{dx} = -2$

$6x^2 + 30x + 34 = -2$, so $6x^2 + 30x + 36 = 0$ ($\div 6$: $x^2 + 5x + 6 = 0$)
$(x + 3)(x + 2) = 0$ so $x = -3$ or $x = -2$
A$(-3, -14)$ and B$(-2, -17)$
Answer: $(-3, -14)$ and $(-2, -17)$

5 Quadratic formula and surds

Solve $x^2 + 8x + 1 = 0$
Give your answers in the form $a \pm \sqrt{b}$

Complete the square: $(x + 4)^2 - 16 + 1 = 0$
$(x + 4)^2 = 15$
$x + 4 = \pm\sqrt{15}$
$x = -4 \pm \sqrt{15}$
Answer: $x = -4 \pm \sqrt{15}$

6 Probability with sets

n(ξ) = 100, n(A) = 27, n(B) = 20 and n(A ∩ B) = 5.
An element of ξ is chosen at random.
(a) Find the probability that it is in neither A nor B.
(b) Given that it is in A, find the probability that it is not in B.

$n(A \cup B) = 27 + 20 - 5 = 42$

(a) $n(A \cup B)' = 100 - 42 = 58$, so $P = \frac{58}{100} = \frac{29}{50}$

(b) In A there are 27 elements, and $27 - 5 = 22$ of them are not in B

$P = \frac{22}{27}$

Answer: (a) $\frac{29}{50}$ (b) $\frac{22}{27}$

7 Stationary points of a quartic

A curve has equation $y = x^4 - 2x^2 + 6$.
Find the coordinates of the stationary points of the curve and determine the nature of each.

$\frac{dy}{dx} = 4x^3 - 4x = 4x(x - 1)(x + 1)$

$\frac{dy}{dx} = 0$ gives $x = 0$, $x = 1$ or $x = -1$

$y = 6$ when $x = 0$, and $y = 5$ when $x = \pm 1$

$\frac{d^2y}{dx^2} = 12x^2 - 4$: it is -4 at $x = 0$ (maximum) and 8 at $x = \pm 1$ (minimum)

Answer: (0, 6) maximum; (1, 5) and (−1, 5) minima

8 Bearings with the sine and cosine rules

A walker leaves a camp P and travels 42 km on a bearing of 335° to Q. (Calculator)
It then travels 94 km on a bearing of 105° to R.
(a) Work out the distance PR.
(b) Work out the bearing of R from P.
Give distances to 1 decimal place and the bearing to the nearest degree.

(a) The bearing of P from Q is 335° − 180° = 155°
Angle PQR = 50° (the difference between the bearings 155° and 105°)
$PR^2 = 42^2 + 94^2 - 2 \times 42 \times 94 \times \cos 50° = 5524.5$, so PR = 74.3 km

(b) $\cos(QPR) = \frac{42^2 + PR^2 - 94^2}{2 \times 42 \times PR}$, so angle QPR = 104.4°

R is clockwise of the direction PQ, so the bearing is 335° + 104° = 079°
Answer: (a) 74.3 km (b) 079°

9 Tree diagrams without replacement

A bag contains 3 red beads and 5 blue beads.
Two beads are taken at random without replacement.
Work out the probability that (a) exactly one is red, (b) at least one is red.

(a) P(exactly one red) = $\frac{3}{8} \times \frac{5}{7} + \frac{5}{8} \times \frac{3}{7} = \frac{15}{28}$

(b) P(at least one red) = 1 − P(none) = $1 - \frac{5}{8} \times \frac{4}{7} = \frac{9}{14}$

Answer: (a) $\frac{15}{28}$ (b) $\frac{9}{14}$

10 Show that the gradient is always positive

A curve has equation $y = x^3 - x^2 + 3x - 5$.
Show that y is an increasing function for all values of x.

$\frac{dy}{dx} = 3x^2 - 2x + 3$

Discriminant $= -2^2 - 4 \times 3 \times 3 = -32$

The discriminant is negative, so $\frac{dy}{dx} = 0$ has no real solutions

The coefficient of x^2 is positive, so $\frac{dy}{dx} > 0$ for all x

So y is increasing for all values of x

Answer: Discriminant $= -32 < 0$, so $dy/dx > 0$ for all x