Day 193 **IGCSE 10-a-day: Model Solutions**

1 Gradient at a point

Work out the gradient of the curve $y = -3x^2 + 7x - 6$ at the point where $x = -3$.

$\frac{dy}{dx} = -6x + 7$

When $x = -3$: gradient = 25

Answer: 25

2 Time zones

A flight leaves London at 22:05 (London time).
It takes 11 hours 30 minutes.
When it is 12:00 in London it is 14:00 in Cairo.
At what time, in Cairo time, does the flight land? Say if it is on a different day.

Arrival in London time = 22:05 + 11 h 30 min = 09:35 the next day
Cairo is 2 hours ahead of London, so add 2 hours
11:35 the next day

Answer: 11:35 the next day

3 Transformations of graphs

The point P(12, 7) lies on the graph of y = f(x).
Write down the coordinates of the image of P on the graph of
(a) y = f(x) + 2 (b) y = f(2x)

(a) y = f(x) + 2 is a translation of 2 up: (12, 9)

(b) y = f(2x) is a horizontal stretch with scale factor $\frac{1}{2}$, so the x coordinate is halved: (6, 7)

Answer: (a) (12, 9) (b) (6, 7)

4 Fractions

Work out $(4\frac{2}{3} - 1\frac{1}{2}) \times \frac{3}{8}$

Give your answer as a mixed number or a fraction in its simplest form.

Change the mixed numbers to improper fractions

$\frac{14}{3} - \frac{3}{2} = \frac{19}{6}$

$\frac{19}{6} \times \frac{3}{8} = \frac{19}{16}$

$= 1\frac{3}{16}$

Answer: $1\frac{3}{16}$

5 Normal in the form ax + by + c = 0

The point A lies on the curve $y = x^2 - 7x + 7$ where $x = 1$.
Find the equation of the normal at A. Give your answer in the form ax + by + c = 0 where a, b and c are integers.

y = 1 at x = 1 and gradient of curve = −5

Gradient of normal = $\frac{1}{5}$

$y - 1 = \frac{1}{5}(x - 1)$

$y = \frac{1}{5}x + \frac{4}{5}$

Multiply by 5: $x - 5y + 4 = 0$

Answer: $x - 5y + 4 = 0$

6 Solve f inverse equals g

$f(x) = \frac{x + 6}{3}$ and $g(x) = x^2 - 4$.

Solve $f^{-1}(x) = g(x)$.

$f^{-1}(x) = 3x - 6$

$3x - 6 = x^2 - 4$

$x^2 - 3x + 2 = 0$

$(x - 1)(x - 2) = 0$

$x = 1$ or $x = 2$

Answer: $x = 1$ or $x = 2$

7 Gradient with fractional or negative powers

The equation of a curve is $y = 4x^2 + \frac{11}{x}$.

Work out the gradient of the curve at the point where $x = 1$.

$\frac{dy}{dx} = 8x - 11x^{-2}$

When $x = 1$: gradient $= -3$

Answer: -3

8 Frustum of a cone

A cone is cut parallel to its base to make a smaller cone and a frustum.
The frustum has height 4 cm, top radius 6 cm and base radius 18 cm.
Work out the volume of the frustum. Give your answer in terms of π.

Let the height of the small cone removed be h cm. The cones are similar, so $\frac{h}{h + 4} = \frac{6}{18}$

18 h = 6(h + 4), so 12 h = 24 and h = 2

Large cone: $\frac{1}{3} \times \pi \times 18^2 \times 6 = 648\pi$

Small cone: $\frac{1}{3} \times \pi \times 6^2 \times 2 = 24\pi$

Frustum $= 648\pi - 24\pi = 624\pi$ cm^3

Answer: 624π cm^3

9 Overall percentage change from repeated changes

The price of a coat is decreased by 50% and then increased by 30%.
Work out the overall percentage change. State whether it is an increase or a decrease.

Multipliers: 0.5 and 1.3
Overall multiplier = 0.5 × 1.3 = 0.65
35% decrease
Answer: 35% decrease

10 Density of an alloy

An alloy is made by mixing 120 cm^3 of lead with 30 cm^3 of nickel.
The density of lead is 11.3 g/cm^3. The density of nickel is 8.9 g/cm^3.
Work out the density of the alloy.

Mass of lead = 120 × 11.3 = 1356 g
Mass of nickel = 30 × 8.9 = 267 g
Total mass = 1623 g, total volume = 150 cm^3
Density = 1623 ÷ 150 = 10.82 g/cm^3
Answer: 10.82 g/cm^3