Day 192 **IGCSE 10-a-day: Model Solutions**

1 Exact trigonometry in triangles

ABC is a right-angled triangle.
Work out the exact value of x. Give your answer in its simplest form.
The diagram is not drawn accurately.

x is the hypotenuse AC and the known side is the opposite side BC, so use sin 60°

$\sin 60° = \frac{\sqrt{3}}{2}$

$x = 4\sqrt{3} \div \sin 60° = 4\sqrt{3} \div \frac{\sqrt{3}}{2}$

x = 8 (after rationalising the denominator)

Answer: x = 8

2 Turning point of a quadratic by differentiation

$f(x) = -2x^2 - 12x + 3$

Use differentiation to find the coordinates of the turning point of the graph of y = f(x). State whether it is a maximum or a minimum.

$f'(x) = -4x - 12$
$f'(x) = 0$: $-4x - 12 = 0$, so $x = -3$
$f(-3) = 21$
$f''(x) = -4$, which is negative, so it is a maximum

Answer: (−3, 21), maximum

3 Weighted percentages

The ratio of Year 10 students to Year 11 students in a school is 1 : 3.
30% of the Year 10 students and 80% of the Year 11 students have a part-time job.
What percentage of all of these students have a part-time job?

Imagine 80 students in total: 20 Year 10 students and 60 Year 11 students
30% of 20 = 6 and 80% of 60 = 48
Together: 54 out of 80
54 ÷ 80 × 100 = 67.5%

Answer: 67.5%

4 Increasing and decreasing intervals

$y = x^3 - 3x^2 - 24x - 9$

Find the range of values of x for which y is decreasing.

$\frac{dy}{dx} = 3x^2 - 6x - 24 = 3(x + 2)(x - 4)$

$\frac{dy}{dx} = 0$ when $x = -2$ and $x = 4$

The graph of $\frac{dy}{dx}$ against x is a U-shaped quadratic

y is increasing when $\frac{dy}{dx} > 0$ and decreasing when $\frac{dy}{dx} < 0$

So y is decreasing when $-2 < x < 4$

Answer: $-2 < x < 4$

5 Transformation by a matrix: image and description

$M = \begin{pmatrix} 1 & 0 \\ 0 & 4 \end{pmatrix}$.

(a) Find the coordinates of the image of the point (3, 3) under the transformation represented by M.

(b) Describe fully the single transformation represented by M.

(a) $\begin{pmatrix} 1 & 0 \\ 0 & 4 \end{pmatrix}\begin{pmatrix} 3 \\ 3 \end{pmatrix} = \begin{pmatrix} 3 \\ 12 \end{pmatrix}$

The image is (3, 12)

(b) The first column is where (1, 0) goes and the second column is where (0, 1) goes

M represents a stretch, scale factor 4, x-axis invariant

Answer: (a) (3, 12) (b) Stretch, scale factor 4, x-axis invariant

6 Equations with indices

Solve $x^{\frac{4}{3}} = 16$

$x > 0$

Write 16 as a power of 2: $16 = 2^4$

$x^{\frac{4}{3}} = 2^4$, so $x = (2^4)^{\frac{3}{4}}$

$x = 2^3 = 8$

Answer: x = 8

7 Fencing against a wall

A farmer uses 56 m of fencing to enclose a rectangle against a wall, as shown.

(a) Show that the area, A m^2, is $A = 56x - 2x^2$.

(b) Find the maximum area, proving it is a maximum.

Side parallel to the wall = $56 - 2x$, so $A = x(56 - 2x) = -2x^2 + 56x$

$\frac{dA}{dx} = -4x + 56 = 0$ gives $x = 14$

$\frac{d^2A}{dx^2} = -4 < 0$, so a maximum

$A = 56 \times 14 - 2 \times 14^2 = 392$

Answer: 392 m^2

8 Estimating the mean from a histogram table

The table gives information about the times of some people.

Work out an estimate for the mean time.

Give your answer correct to 1 decimal place. (Calculator)

Frequency = frequency density × class width: 4, 9, 9, 4

Mid-points: 5, 17.5, 32.5, 50

$\Sigma fm = 4\times5 + 9\times17.5 + 9\times32.5 + 4\times50 = 670$, $\Sigma f = 26$

Mean = 670 ÷ 26 = 25.8

Answer: 25.8

9 Bounds of a compound expression

p = 9.2 and r = 6, each correct to 1 decimal place.

q = 3.9, correct to 2 decimal places.

Work out the upper bound of $\frac{p - q}{r}$. Give your answer correct to 3 significant figures. (Calculator)

For the upper bound use UB of the top and LB of the bottom

Top: UB p − LB q = 9.25 − 3.895 = 5.355

Bottom: LB r = 5.95

5.355 ÷ 5.95 = 0.900

Answer: 0.900

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10 Nested surds

Show that $\sqrt{70 + 24\sqrt{6}} = 4 + 3\sqrt{6}$

Square the right-hand side: $(4 + 3\sqrt{6})^2 = 16 + 24\sqrt{6} + 54$

$= 70 + 24\sqrt{6}$

So $\sqrt{70 + 24\sqrt{6}} = 4 + 3\sqrt{6}$

Answer: $\boldsymbol{\sqrt{70 + 24\sqrt{6}} = 4 + 3\sqrt{6}}$