Day 191 **IGCSE 10-a-day: Model Solutions**

1 Ordering numbers

Write these numbers in order of size, starting with the smallest.

95%, $\frac{2}{3}$, $\frac{7}{20}$, 0.83

Convert each to a decimal: 95% = 0.95, $\frac{2}{3}$ = 0.667, $\frac{7}{20}$ = 0.35, 0.83 = 0.83

In order: $\frac{7}{20}$, $\frac{2}{3}$, 0.83, 95%

Answer: $\frac{7}{20}$, $\frac{2}{3}$, 0.83, 95%

2 Expand then differentiate

$y = (x + 2)(x + 3)$

Find $\frac{dy}{dx}$.

Expand: $y = x^2 + 5x + 6$

$x^2 \rightarrow 2x$, $5x \rightarrow 5$

$\frac{dy}{dx} = 2x + 5$

Answer: $\frac{dy}{dx} = 2x + 5$

3 Pressure

A concrete slab in the shape of a cuboid weighs 210 N.

Its smallest face has area 250 cm^2. Its largest face has area 1,500 cm^2.

Pressure = force ÷ area

Work out (a) the greatest pressure and (b) the least pressure the concrete slab can exert on the ground, in N/m^2.

1 m^2 = 10 000 cm^2, so 250 cm^2 = 0.025 m^2 and 1,500 cm^2 = 0.15 m^2

(a) Greatest pressure is on the smallest face: 210 ÷ 0.025 = 8,400 N/m^2

(b) Least pressure is on the largest face: 210 ÷ 0.15 = 1,400 N/m^2

Answer: (a) 8,400 N/m^2 (b) 1,400 N/m^2

4 Intersection and gradient

The curve $y = -x^2 + 5x + 2$ and the line $y = 8x + 4$ intersect at two points A and B, where A has the smaller x coordinate.

(a) Find the coordinates of A and B.

(b) Find the gradient of the curve at A and at B.

$-x^2 + 5x + 2 = 8x + 4$ gives $-x^2 - 3x - 2 = 0$, so $-1(x + 2)(x + 1) = 0$

$x = -2$ or $x = -1$, giving A $(-2, -12)$ and B $(-1, -4)$

$\frac{dy}{dx} = -2x + 5$: at A it is 9, at B it is 7

Answer: (a) A $(-2, -12)$, B $(-1, -4)$ (b) 9 at A, 7 at B

5 Equation of a curve from a sketch

The diagram shows a sketch of a quadratic curve with a turning point at (2, 7).

The curve crosses the y-axis at (0, 3).

Find an equation of the curve.

Give your answer in the form $y = ax^2 + bx + c$.

The turning point is (2, 7), so $y = a(x - 2)^2 + 7$

When x = 0, y = 3: $a \times 4 + 7 = 3$, so $a = -1$

$y = -(x - 2)^2 + 7$

Expand: $y = -x^2 + 4x + 3$

Answer: $y = -x^2 + 4x + 3$

6 Differentiating brackets with square roots

$y = \sqrt{x}(x + 2)$

Find $\frac{dy}{dx}$. Give each term of your answer in the form ax^n.

Expand: $y = x^{3/2} + 2x^{1/2}$

$x^{3/2} \rightarrow \frac{3}{2}x^{1/2}$, $2x^{1/2} \rightarrow x^{-1/2}$

$\frac{dy}{dx} = \frac{3}{2}x^{1/2} + x^{-1/2}$

Answer: $\mathbf{\frac{dy}{dx} = \frac{3}{2}x^{1/2} + x^{-1/2}}$

7 Three picks without replacement

A bag has 4 blue and 6 red beads.
Three beads are taken at random without replacement.
(a) Work out the probability that all three are red.
(b) Work out the probability that at least one is blue.

(a) $\frac{6}{10} \times \frac{5}{9} \times \frac{4}{8} = \frac{1}{6}$

(b) At least one blue is the opposite of all three red

$1 - \frac{1}{6} = \frac{5}{6}$

Answer: (a) $\mathbf{\frac{1}{6}}$ **(b)** $\mathbf{\frac{5}{6}}$

8 Pythagoras with algebra

ABC is a right-angled triangle with sides (x + 8) cm, (x + 1) cm and (2x + 3) cm. The longest side is (2x + 3) cm.
(a) Find the value of x.
(b) Work out the area of the triangle.
The diagram is not drawn accurately.

(a) Pythagoras: $(x + 8)^2 + (x + 1)^2 = (2x + 3)^2$
Expand and collect terms: $2x^2 - 6x - 56 = 0$
Factorise: $2(x - 7)(x + 4) = 0$
$x = 7$ or $x = -4$, and x must be positive, so $x = 7$
(b) The two shorter sides are 15 cm and 8 cm

Area $= \frac{1}{2} \times 15 \times 8 = 60$ cm^2

Answer: (a) x = 7 (b) 60 cm^2

9 Histogram with unequal class widths

The histogram shows the ages, in years, of some visitors.
The bar for 35 to 40 represents 30 visitors.
Estimate the number of visitors with a value greater than 36.

Frequency = k × height × width. $30 = k \times 6 \times 5$ so $k = 1$
Part of the 35 to 40 bar: $1 \times 6 \times (40 - 36) = 24$
Bars above 40: 35
Total = 59
Answer: 59

10 Density of an alloy

An alloy is made by mixing 20 cm^3 of nickel with 100 cm^3 of lead.
The density of nickel is 8.9 g/cm^3. The density of lead is 11.3 g/cm^3.
Work out the density of the alloy.

Mass of nickel = 20 × 8.9 = 178 g
Mass of lead = 100 × 11.3 = 1130 g
Total mass = 1308 g, total volume = 120 cm^3
Density = 1308 ÷ 120 = 10.9 g/cm^3
Answer: 10.9 g/cm^3