Day 190 **IGCSE 10-a-day: Model Solutions**

1 Time zones

A flight leaves London at 06:40 (London time).
It takes 9 hours 45 minutes.
When it is 12:00 in London it is 19:00 in Singapore.
At what time, in Singapore time, does the flight land? Say if it is on a different day.

Arrival in London time = 06:40 + 9 h 45 min = 16:25
Singapore is 7 hours ahead of London, so add 7 hours
23:25
Answer: 23:25 the same day

2 Describe a graph transformation

Describe fully the single transformation that maps the graph of y = f(x) onto the graph of
(a) y = 3f(x)
(b) y = −f(x)

(a) Every y value is multiplied by 3
(b) Changing the sign of every y value flips the graph over the x-axis
Answer: (a) A stretch parallel to the y-axis with scale factor 3 (b) A reflection in the x-axis

3 Equation of a tangent

A curve has equation $y = -x^2 + 4x - 5$.
Find the equation of the tangent to the curve at the point where $x = -3$.

When $x = -3$, $y = -26$
$\frac{dy}{dx} = -2x + 4$, so the gradient at $x = -3$ is 10
$y + 26 = 10(x + 3)$
$y = 10x + 4$
Answer: $y = 10x + 4$

4 Venn diagram: shaded region notation

Write down the set notation for the region shaded in the Venn diagram.

Identify which regions are shaded, using the circles P, Q and R
The shaded region is $Q \cap R \cap P$
Answer: $Q \cap R \cap P$

5 Tangent to a factorised cubic

A curve has equation $y = (x + 3)(x - 3)(x + 3)$.
Find the equation of the tangent to the curve at the point where $x = 2$.

Expand: $y = x^3 + 3x^2 - 9x - 27$
$\frac{dy}{dx} = 3x^2 + 6x - 9$
At $x = 2$: $y = -25$ and gradient = 15
$y + 25 = 15(x - 2)$
$y = 15x - 55$
Answer: $y = 15x - 55$

6 Successive percentage change

The value of a car is decreased by 40%.
The new amount is then decreased by 15%.
Work out the overall percentage change.

Multipliers: 0.6 and 0.85
0.6 × 0.85 = 0.51
0.51 is a decrease of 49%
Answer: 49% decrease

7 Find constants from a turning point

The diagram shows a sketch of the curve $y = ax^2 + bx + 6$.
The curve has a turning point at M(−2, −2).
Find the values of a and b.

$\frac{dy}{dx} = 2ax + b = 0$ when $x = -2$, so $b = 4a$
The point (−2, −2) is on the curve: $4a - 2b + 6 = -2$
Substitute $b = 4a$: $-4a = -8$, so $a = 2$ and $b = 8$
Answer: a = 2, b = 8

8 Comparing box plots

The box plots show the test marks of the students in two classes, A and B.
(a) Work out the interquartile range for each.
(b) Compare the distributions of A and B.

(a) IQR of A = 80 − 40 = 40
IQR of B = 50 − 20 = 30
(b) Class A has the higher median (75 compared with 35), so on average Class A scored higher
Class B has the smaller interquartile range (30 compared with 40), so its marks are more consistent
Answer: (a) A: 40, B: 30 (b) A has the higher median, B has the smaller IQR

9 Where a line meets the axes

The line L has equation $3x - 4y = -24$.
L crosses the x-axis at the point A and the y-axis at the point B.
(a) Find the coordinates of A and B.
(b) Work out the length of AB.

(a) At A, y = 0: $3x = -24$, so $x = -8$. A is (−8, 0)
At B, x = 0: $-4y = -24$, so $y = 6$. B is (0, 6)
(b) $AB^2 = 8^2 + 6^2 = 100$
$AB = \sqrt{100} = 10$
Answer: (a) A(−8, 0), B(0, 6) (b) 10

10 Tangent to a circle

A circle has equation $x^2 + y^2 = 169$.
The point P(−12, −5) lies on the circle.
(a) Find the equation of the tangent to the circle at P. Give your answer in the form ax + by = c, where a, b and c are integers.
(b) The tangent meets the x-axis at the point T. Write down the coordinates of T.

(a) The gradient of the radius OP is $\frac{5}{12}$

The tangent is perpendicular to the radius, so its gradient is $-1 \div \frac{5}{12} = -\frac{12}{5}$

$y - (-5) = (-\frac{12}{5})(x - (-12))$

Multiply out and rearrange: $12x + 5y = -169$
(b) On the x-axis y = 0, so $12x = -169$
$x = -\frac{169}{12}$, so T is $(-\frac{169}{12}, 0)$

Answer: (a) 12x + 5y = −169 (b) T = $(-\frac{169}{12}, 0)$