Day 19 **IGCSE 10-a-day: Model Solutions**

1 Solve f′(x) = 0 with a reciprocal term

$f(x) = x^3 + \frac{1875}{x}$

Find the values of x for which f′(x) = 0.

$f'(x) = 3x^2 - \frac{1875}{x^2}$

$f'(x) = 0$: $3x^2 = \frac{1875}{x^2}$

$x^4 = 625$

$x = 5$ or $x = -5$

Answer: $x = 5$ or $x = -5$

2 Angles with algebra

ABC is an isosceles triangle with AB = AC.
Angle BAC = (4x)°
Angle ABC = (x + 27)°
(a) Work out the value of x.
(b) Work out the size of angle BAC.

(a) Angles ABC and ACB are equal, and the angles in a triangle add up to 180°
(4x) + 2(x + 27) = 180
6x + 54 = 180, so 6x = 126
x = 21
(b) Angle BAC = 4 × 21 + 0 = 84°
Answer: (a) $x = 21$ (b) 84°

3 Inverse of a rational function

$f(x) = \frac{-3x + 2}{3x + 3}$, $x \neq -1$.

Find $f^{-1}(x)$.

$y = \frac{-3x + 2}{3x + 3}$, so $y(3x + 3) = -3x + 2$

$3xy + 3y = -3x + 2$

$3xy + 3x = 2 - 3y$

$x(3y + 3) = 2 - 3y$, so $x = \frac{2 - 3y}{3y + 3}$

$f^{-1}(x) = \frac{2 - 3x}{3x + 3}$

Answer: $f^{-1}(x) = \frac{2 - 3x}{3x + 3}$

4 Trigonometric graphs

The diagram shows the graph of $y = a \sin x° + c$ for $0 \leq x \leq 360$.
Some points on the graph are labelled.
Find the value of a and the value of c.

At x = 90, sin x° = 1, so $a + c = -4$
At x = 270, sin x° = −1, so $-a + c = -10$
Add: $2c = -14$, so $c = -7$
Subtract: $2a = 6$, so $a = 3$
Answer: $a = 3$, $c = -7$

5 Compound interest

Isla invests £4,500 for 5 years at 3.5% per year compound interest.
Work out the value of the investment at the end of the 5 years.
Give your answer to the nearest penny. (Calculator)

Multiplier = $1 + 3.5 \div 100 = 1.035$

$4500 \times 1.035^5 = 5344.59$

To the nearest penny: £5,344.59

Answer: £5,344.59

6 Curve touching the x-axis

The curve $y = -x^3 + 9x^2 - 24x + k$, where k is a constant, has the x-axis as a tangent.
Find the two possible values of k.

$\frac{dy}{dx} = -3x^2 + 18x - 24 = 0$ gives x = 2 or x = 4

The x-axis is a tangent when a turning point has y = 0

x = 2: $-20 + k = 0$, so k = 20

x = 4: $-16 + k = 0$, so k = 16

Answer: k = 20 or k = 16

7 Exact trigonometric ratios from a given ratio

x is an acute angle and $\tan x = \frac{8}{13}$.

(a) Find the exact value of sin x.

(b) Find the exact value of cos x. Give each answer as a fraction or a surd with a rational denominator.

Draw a right-angled triangle with opposite side 8 and adjacent side 13

Hypotenuse = $\sqrt{8^2 + 13^2} = \sqrt{233}$

(a) $\sin x = \frac{8}{\sqrt{233}} = \frac{8\sqrt{233}}{233}$

(b) $\cos x = \frac{13}{\sqrt{233}} = \frac{13\sqrt{233}}{233}$

Answer: (a) $\frac{8\sqrt{233}}{233}$ (b) $\frac{13\sqrt{233}}{233}$

8 Increasing range for y = ax + b/x

$y = 3x + \frac{48}{x}$

Find the values of x for which y is increasing.

$\frac{dy}{dx} = 3 - \frac{48}{x^2}$

$\frac{dy}{dx} > 0$: $3 > \frac{48}{x^2}$

$x^2 > 16$ (x^2 is positive, so multiply through)

$x < -4$ or $x > 4$

Answer: $x < -4$ or $x > 4$

9 Expanding brackets with surds

Expand and simplify $(2 + 2\sqrt{5})(1 - 2\sqrt{5})$

Give your answer in the form $p + q\sqrt{5}$

$(2 + 2\sqrt{5})(1 - 2\sqrt{5}) = 2 - 4\sqrt{5} + 2\sqrt{5} - 20$

$= -18 - 2\sqrt{5}$

Answer: $-18 - 2\sqrt{5}$

10 Self-inverse function

$f(x) = \frac{-5x - 3}{x - k}$, where k is a constant and $x \neq k$.

The function f is its own inverse, so $f^{-1}(x) = f(x)$.
Find the value of k.

Let $y = \frac{-5x - 3}{x - k}$, so $y(x - k) = -5x - 3$

$xy - ky = -5x - 3$, so $x(y + 5) = ky - 3$

$f^{-1}(x) = \frac{kx - 3}{x + 5}$

For $f^{-1}(x) = f(x)$ the denominators must match: $x + 5 = x - k$

Answer: $k = -5$