Day 189 **IGCSE 10-a-day: Model Solutions**

1 Fastest rate of growth

The number of fish in a lake, N, after t months is modelled by

$N = -t^3 + 9t^2 + 6t + 500$ for $0 \leq t \leq 12$

Find the time at which N is increasing most rapidly, and the rate of increase at this time.

$\frac{dN}{dt} = -3t^2 + 18t + 6$

Rate is greatest when $\frac{d^2N}{dt^2} = -6t + 18 = 0$, so $t = 3$

$\frac{d^2N}{dt^2}$ is decreasing (gradient -6), so the rate is a maximum

$\frac{dN}{dt} = 33$ when $t = 3$

Answer: t = 3, rate 33 per month

2 Time zones

A flight leaves London at 12:55 (London time).

It takes 9 hours 35 minutes.

When it is 12:00 in London it is 14:00 in Cairo.

At what time, in Cairo time, does the flight land? Say if it is on a different day.

Arrival in London time = 12:55 + 9 h 35 min = 22:30

Cairo is 2 hours ahead of London, so add 2 hours

00:30 the next day

Answer: 00:30 the next day

3 Gradient from a tangent

The diagram shows part of the curve $y = -2x^2 - 5x - 4$. The line shown is the tangent to the curve at the point P. (Not drawn accurately.)

(a) Use the two marked points on the tangent to work out its gradient.

(b) Use differentiation to find the x coordinate of P.

Gradient = $\frac{1 - (-8)}{-1 - (-4)} = 3$

$\frac{dy}{dx} = -4x - 5$

$-4x - 5 = 3$, so $x = -2$

Answer: (a) 3 (b) x = −2

4 Hollow cylinder and density

A metal pipe is a hollow cylinder of length 30 cm. (Calculator)

The outer radius is 10 cm and the inner radius is 8 cm.

The density of the metal is 8.9 g/cm^3.

Work out the mass of the pipe. Give your answer in kilograms correct to 3 significant figures.

Volume of metal = $\pi \times 10^2 \times 30 - \pi \times 8^2 \times 30 = \pi(100 - 64) \times 30$

= 3392.9 cm^3

Mass = density × volume = 8.9 × 3392.9 = 30197 g

= 30.2 kg (3 s.f.)

Answer: 30.2 kg

5 Compound interest: how many years

£8,000 is invested at 5% per year compound interest.

Work out the least whole number of years after which the investment is worth more than £12,000. (Calculator)

Value after n years = 8000×1.05^n

After 8 years: £11,819.64 (less than £12,000)

After 9 years: £12,410.63 (more than £12,000)

Answer: 9 years

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6 Laws of indices

Given that $\frac{16^5 \times 2^4}{4^2} = 2^n$,

find the value of n.

$16 = 2^4$, $4 = 2^2$

Numerator: $2^{20} \times 2^4 = 2^{24}$

Denominator: 2^4

Subtract the powers: $24 - 4 = 20$

Answer: n = 20

7 Equations in disguise

Solve $x - 11\sqrt{x} + 28 = 0$

Let $u = \sqrt{x}$: $u^2 - 11u + 28 = 0$

Factorise: $(u - 7)(u - 4) = 0$, so $u = 7$ or $u = 4$

$\sqrt{x} = 7$ or $\sqrt{x} = 4$

$x = 49$ or $x = 16$

Answer: x = 16 or x = 49

8 3D trigonometry: pyramid

VABCD is a pyramid. The base ABCD is a square with sides of 9 cm.
The vertex V is 9 cm vertically above the centre O of the base.
(a) Work out the length of AC.
(b) Work out the angle between VA and the base. (Calculator)

(a) $AC^2 = 9^2 + 9^2 = 162$, AC = 12.73 cm

(b) AO = ½AC = 6.364 cm

$\tan VAO = \frac{VO}{AO} = \frac{9}{6.364}$

Angle = 54.7°

Answer: (a) 12.7 cm (b) 54.7°

9 Show that a line is a tangent

The line $y = 2x - 4$ meets the curve $y = 2x^2 - 2x - 2$.
Show that the line is a tangent to the curve and find the coordinates of the point where they touch.

Equate: $2x^2 - 2x - 2 = 2x - 4$, so $2x^2 - 4x + 2 = 0$

Discriminant: $(-4)^2 - 4 \times 2 \times 2 = 16 - 16 = 0$

One repeated root, so the line touches the curve at one point only: it is a tangent

$x = \frac{-(-4)}{2 \times 2} = 1$ and $y = -2$

Answer: (1, −2)

10 Combining means

The mean of the masses in kg for the boys is 57.
The mean of the masses in kg for the girls is 59.
The mean for all of the boys and girls together is 57.8.
Work out the ratio of the number of boys to the number of girls.

Let there be p boys and q girls

Total = 57p + 59q, and the overall mean gives $57p + 59q = 57.8(p + q)$

$-0.8p = -1.2q$

$p : q = -1.2 : -0.8 = 3 : 2$

Answer: 3 : 2