Day 188 **IGCSE 10-a-day: Model Solutions**

1 Laws of indices with algebra

Simplify fully $\frac{6x^{\frac{3}{2}} \times 2x^{\frac{4}{3}}}{6x^{\frac{5}{2}}}$

Numbers: $6 \times 2 \div 6 = 2$

Add the powers for × and subtract for ÷: $\frac{3}{2} + \frac{4}{3} - (\frac{5}{2}) = \frac{1}{3}$

$= 2x^{\frac{1}{3}}$

Answer: $2x^{\frac{1}{3}}$

2 Negative powers

$y = 5x^2 + 5x - \frac{1}{x^2}$

Find $\frac{dy}{dx}$.

Write negative powers as indices: $y = 5x^2 + 5x - x^{-2}$

$5x^2 \rightarrow 10x$, $5x \rightarrow 5$, $-x^{-2} \rightarrow 2x^{-3}$

$\frac{dy}{dx} = 10x + 5 + 2x^{-3}$

Answer: $\frac{dy}{dx} = 10x + 5 + \frac{2}{x^3}$

3 Sine rule

The diagram shows triangle ABC. (Calculator)
Work out the length of AC. Give your answer correct to 1 decimal place.
The diagram is not drawn accurately.

Use the sine rule: $\frac{AC}{\sin B} = \frac{BC}{\sin A}$

$\frac{x}{\sin 54°} = \frac{13}{\sin 60°}$

$x = \frac{13 \times \sin 54°}{\sin 60°} = 12.144$

x = 12.1 cm (1 d.p.)

Answer: AC = 12.1 cm

4 Fraction problems

A bag contains balloons.

$\frac{2}{3}$ of the balloons are pink and $\frac{1}{5}$ of the balloons are silver.

The rest of the balloons, 8 of them, are gold.

How many pink balloons are there?

Fraction that are gold = $1 - \frac{2}{3} - \frac{1}{5} = \frac{2}{15}$

$\frac{2}{15}$ of the total is 8

Total = $8 \div \frac{2}{15} = 60$

Pink balloons = $\frac{2}{3}$ of 60 = 40

Answer: 40

5 Geometric sequences

The first three terms of a geometric sequence are
3, −9, 27
(a) Write down the common ratio.
(b) Work out the 7th term.

(a) $-9 \div 3 = -3$

(b) nth term = $3 \times (-3)^{n-1}$

7th term = $3 \times (-3)^6 = 2187$

Answer: (a) −3 (b) 2187

6 Normal meets the x-axis

The normal to the curve $y = -x^2 + 3x + 4$ at the point where $x = -2$ meets the x-axis at N.
Find the coordinates of N.

$y = -6$ and gradient of curve = 7

Gradient of normal = $-\frac{1}{7}$

$y + 6 = -\frac{1}{7}(x + 2)$

Put $y = 0$: $6 = -\frac{1}{7}(x + 2)$

$x = -44$

Answer: (−44, 0)

7 Three-part ratios

Evan, Nadia and Leila share some money.
Evan : Nadia = 3 : 6
Nadia : Leila = 5 : 6
(a) Work out the ratio Evan : Nadia : Leila.
(b) Leila gets £28 more than Evan. How much money does Nadia get?

(a) Make the Nadia part the same: 3 : 6 = 15 : 30 and 5 : 6 = 30 : 36
Evan : Nadia : Leila = 15 : 30 : 36 = 5 : 10 : 12
(b) Difference in parts = 7, so 1 part = 28 ÷ 7 = 4
Nadia gets 10 × 4 = £40
Answer: (a) 5 : 10 : 12 (b) £40

8 Open box from a square sheet

Squares of side x cm are cut from the corners of a 54 cm square plastic sheet, which is folded into an open box of volume V cm^3.
(a) Show that $V = 4x^3 - 216x^2 + 2916x$.
(b) Find x for the maximum volume, and this volume.

Base is a square of side $54 - 2x$, so $V = x(54 - 2x)^2 = x(2916 - 216x + 4x^2) = 4x^3 - 216x^2 + 2916x$

$\frac{dV}{dx} = 12x^2 - 432x + 2916 = 0$, so $x^2 - 36x + 243 = 0$

$12(x - 9)(x - 27) = 0$, so $x = 9$ or $x = 27$

$x = 27$ is rejected (27 is more than half of 54). $\frac{d^2V}{dx^2} = 24x - 432$, which is -216 when $x = 9$, so a maximum

$V = 11664$

Answer: x = 9, V = 11664 cm^3

9 Histograms with an unmarked scale

The histogram shows the masses of parcels delivered by a courier.
60 of them had a mass between 20 and 40 kg.
(a) How many had a mass between 40 and 50 kg?
(b) How many parcels are there in total?

Frequency is proportional to the area of a bar
20 to 40: bar is 3 squares high and 20 wide, area 60, which represents 60
So 1 unit of area represents 60 ÷ 60 = 1
(a) 40 to 50: 7 × 10 = 70 units of area, so 70 × 1 = 70
(b) Total area = 40 + 60 + 70 + 100 + 60 = 330, so total = 330 × 1 = 330
Answer: (a) 70 (b) 330

10 Inverse of a rational function

$f(x) = \frac{2x + 1}{4x + 1}$, $x \neq -\frac{1}{4}$.

Find $f^{-1}(x)$.

$y = \frac{2x + 1}{4x + 1}$, so $y(4x + 1) = 2x + 1$

$4xy + y = 2x + 1$

$4xy - 2x = 1 - y$

$x(4y - 2) = 1 - y$, so $x = \frac{1 - y}{4y - 2}$

$f^{-1}(x) = \frac{1 - x}{4x - 2}$

Answer: $\mathbf{f^{-1}(x) = \frac{1 - x}{4x - 2}}$