Day 187 **IGCSE 10-a-day: Model Solutions**

1 Normal meets the x-axis

The normal to the curve $y = -3x^2 - 5x + 3$ at the point where $x = -1$ meets the x-axis at N.
Find the coordinates of N.

$y = 5$ and gradient of curve $= 1$
Gradient of normal $= -1$
$y - 5 = -(x + 1)$
Put $y = 0$: $-5 = -(x + 1)$
$x = 4$
Answer: (4, 0)

2 Alternate segment theorem

PAQ is a tangent to the circle at A. B and C are points on the circle. Angle PAB $= (2x - 5)°$, angle ACB $= (3x - 27)°$ and angle CAQ $= 59°$.
(a) Work out the value of x.
(b) Work out the size of angle BAC.
The diagram is not drawn accurately.

(a) Angle ACB = angle PAB (alternate segment theorem)
$3x - 27 = 2x - 5$
$x = 22$
(b) Angle PAB $= 39°$
Angles on the straight line PAQ: angle BAC $= 180 - 39 - 59 = 82°$
Answer: (a) x = 22 (b) 82°

3 Compound interest

£8,000 is invested for 4 years at 3% per year.
Work out how much more interest is earned with compound interest than with simple interest. (Calculator)

Simple interest $= 8000 \times 3 \div 100 \times 4 =$ £960
Compound: $8000 \times 1.03^4 =$ £9,004.07, so interest = £1,004.07
Difference = £1,004.07 − £960 = £44.07
Answer: £44.07

4 Increasing or decreasing at a point

$y = x^3 - 3x^2 - 9x + 8$
At the point where $x = -2$, is the curve increasing or decreasing? You must show your working.

$\frac{dy}{dx} = 3x^2 - 6x - 9$

When $x = -2$: $\frac{dy}{dx} = 15$

15 is positive, so the curve is increasing
Answer: increasing (gradient 15)

5 Fractional indices

Given that $x > 0$, solve $x^{-\frac{3}{2}} = \frac{1}{8}$

Raise both sides to the power $-\frac{2}{3}$

$x = (\frac{1}{8})^{-\frac{2}{3}}$

$x = 4$
Answer: x = 4

6 Same gradient on two curves

Two curves have equations $y = 4x^2 + 5x$ and $y = x^2 + 29x - 8$.
Find the value of x at which the curves have the same gradient.

First curve: $\frac{dy}{dx} = 8x + 5$

Second curve: $\frac{dy}{dx} = 2x + 29$

Equal gradients: $8x + 5 = 2x + 29$
$6x = 24$, so $x = 4$
Answer: x = 4

7 Frustum of a cone

A cone is cut parallel to its base to make a smaller cone and a frustum.
The frustum has height 18 cm, top radius 2 cm and base radius 6 cm.
Work out the volume of the frustum. Give your answer in terms of π.

Let the height of the small cone removed be h cm. The cones are similar, so $\frac{h}{h + 18} = \frac{2}{6}$

$6h = 2(h + 18)$, so $4h = 36$ and $h = 9$

Large cone: $\frac{1}{3} \times \pi \times 6^2 \times 27 = 324\pi$

Small cone: $\frac{1}{3} \times \pi \times 2^2 \times 9 = 12\pi$

Frustum = $324\pi - 12\pi = 312\pi$ cm^3
Answer: 312π cm^3

8 Histograms

The histogram shows the ages of visitors to a museum.
(a) Work out the total number of visitors.
(b) Estimate the number of visitors with an age between 5 years and 15 years.

Frequency = frequency density × class width
0 to 10: 1.2 × 10 = 12
10 to 20: 0.6 × 10 = 6
20 to 30: 0.8 × 10 = 8
30 to 50: 2 × 20 = 40
(a) Total = 66
(b) 5 to 10: 1.2 × 5 = 6
10 to 15: 0.6 × 5 = 3, so estimate = 9
Answer: (a) 66 (b) 9

9 Exact values from a given ratio

x is an acute angle and $\tan x = \frac{5}{8}$.

Find the exact value of (a) sin x (b) cos x.
Give each answer as a fraction, rationalising any denominator.

Right-angled triangle: opposite = 5, adjacent = 8

Hypotenuse = $\sqrt{5^2 + 8^2} = \sqrt{89}$

(a) $\sin x = \frac{5}{\sqrt{89}} = \frac{5\sqrt{89}}{89}$

(b) $\cos x = \frac{8}{\sqrt{89}} = \frac{8\sqrt{89}}{89}$

Answer: (a) $\frac{5\sqrt{89}}{89}$ (b) $\frac{8\sqrt{89}}{89}$

10 Quadratic graphs

The curve $y = -x^2 - 6x + 7$ crosses the x-axis at the points A and B.
(a) Find the coordinates of A and B.
(b) Find the coordinates of the maximum point of the curve.

(a) Put $y = 0$. $-x^2 - 6x + 7 = 0$, so $-(x^2 - 6x - 7) = 0$, giving $-(x + 7)(x - 1) = 0$
$x + 7 = 0$ or $x - 1 = 0$, so $x = -7$ or $x = 1$
(b) The turning point is halfway between A and B: $x = (-7 + 1) \div 2 = -3$
When $x = -3$: $y = -1 \times (-3)^2 - 6 \times (-3) + 7 = 16$
Answer: (a) A(−7, 0), B(1, 0) (b) (−3, 16)