Day 186 **IGCSE 10-a-day: Model Solutions**

1 Quadratic formula and surds

Solve $3x^2 + 3x - 2 = 0$ (Calculator)
Give your answers correct to 2 decimal places.

Use the quadratic formula with a = 3, b = 3, c = −2

$$x = \frac{-3 \pm \sqrt{3^2 - 4 \times 3 \times (-2)}}{2 \times 3} = \frac{-3 \pm \sqrt{33}}{6}$$

x = −1.4574... or x = 0.4574...
x = −1.46 or x = 0.46 (2 d.p.)
Answer: x = −1.46 or x = 0.46

2 Stratified sampling

The table shows the number of members in each of four groups.
(a) A stratified sample of 31 is taken. Work out how many come from Group D.
(b) A different stratified sample has 15 from Group B. Work out the size of this sample.

Total = 350 + 250 + 400 + 550 = 1550

(a) Group D: $\frac{550}{1550} \times 31 = 11$

(b) Fraction sampled = 15 ÷ 250 = $\frac{3}{50}$, so sample size = $\frac{3}{50} \times 1550 = 93$

Answer: (a) 11 (b) 93

3 Tangents and circle theorems

PA and PB are tangents to a circle, centre O. C is a point on the circle. Angle ACB = 42°.
(a) Work out the size of angle AOB.
(b) Work out the size of angle APB.
The diagram is not drawn accurately.

(a) The angle at the centre is twice the angle at the circumference: 2 × 42 = 84°
(b) Angles OAP and OBP are 90° (a tangent meets a radius at 90°)
Angle APB = 360 − 90 − 90 − 84 = 96°
Answer: (a) 84° (b) 96°

4 Determinant and inverse

$M = \begin{pmatrix} 5 & 5 \\ 1 & 3 \end{pmatrix}$.
(a) Work out the determinant of M.
(b) Find M^{-1}.

(a) det M = 5 × 3 − 5 × 1 = 10
(b) Swap the leading diagonal, change the signs of the other two entries, divide by the determinant

$$M^{-1} = \frac{1}{10}\begin{pmatrix} 3 & -5 \\ -1 & 5 \end{pmatrix}$$

Answer: (a) 10 (b) $\frac{1}{10}\begin{pmatrix} 3 & -5 \\ -1 & 5 \end{pmatrix}$

5 Column vectors

$\mathbf{a} = \begin{pmatrix} 4 \\ 3 \end{pmatrix}$ and $\mathbf{b} = \begin{pmatrix} 2 \\ -5 \end{pmatrix}$
Work out the exact length of the vector $\mathbf{a} - \mathbf{b}$. Give your answer as a simplified surd.

$$\mathbf{a} - \mathbf{b} = \begin{pmatrix} 4 \\ 3 \end{pmatrix} - \begin{pmatrix} 2 \\ -5 \end{pmatrix} = \begin{pmatrix} 2 \\ 8 \end{pmatrix}$$

Length = $\sqrt{(2)^2 + (8)^2} = \sqrt{68}$
$= 2\sqrt{17}$
Answer: $2\sqrt{17}$

6 Maximum profit

A company makes and sells x thousand items. All amounts are in thousands of pounds.
The revenue is R = x(29 − 2x) and the cost is C = 22 + 9x.
(a) Show that the profit P = R − C is P = $-2x^2 + 20x - 22$.
(b) Find the number of items that must be sold for the profit to be a maximum, and this maximum profit.

$P = 29x - 2x^2 - (22 + 9x) = -2x^2 + 20x - 22$

$\frac{dP}{dx} = -4x + 20 = 0$ gives $x = 5$

$\frac{d^2P}{dx^2} = -4 < 0$, so a maximum

$P = 28$

Answer: 5000 items, maximum profit £28000

7 Surds with unknowns

Given that $(a + \sqrt{10})^2 = 26 + 8\sqrt{10}$,
where a is a positive integer, find the value of a.

$(a + \sqrt{10})^2 = a^2 + 2a\sqrt{10} + 10$

Compare the $\sqrt{10}$ terms: $2a = 8$, so $a = 4$

Check the integer terms: $a^2 + 10 = 16 + 10 = 26$

Answer: a = 4

8 Tangent parallel to a chord

The diagram shows part of the curve $y = 2x^2 - 2x + 1$. A and B are the points on the curve with x coordinates 0 and 4.
C is the point on the curve between A and B at which the tangent is parallel to AB. Find the coordinates of C.

A = (0, 1) and B = (4, 25)

Gradient of AB = $\frac{25 - 1}{4 - 0} = 6$

$\frac{dy}{dx} = 4x - 2 = 6$, so $x = 2$

$y = 5$

Answer: C (2, 5)

9 Self-inverse function

$f(x) = \frac{-4x - 7}{x - k}$, where k is a constant and $x \neq k$.

The function f is its own inverse, so $f^{-1}(x) = f(x)$.
Find the value of k.

Let $y = \frac{-4x - 7}{x - k}$, so $y(x - k) = -4x - 7$

$xy - ky = -4x - 7$, so $x(y + 4) = ky - 7$

$f^{-1}(x) = \frac{kx - 7}{x + 4}$

For $f^{-1}(x) = f(x)$ the denominators must match: $x + 4 = x - k$

Answer: k = −4

10 Normal to y = kx^2: show that

P is the point (a, $\frac{3}{2}a^2$) on the curve y = $\frac{3}{2}x^2$, where a ≠ 0.

The normal to the curve at P meets the y-axis at the point Q.

Show that the y coordinate of Q is $\frac{3}{2}a^2 + \frac{1}{3}$.

$\frac{dy}{dx}$ = 3x, so the gradient of the tangent at P is 3a

Normal gradient = $-\frac{1}{3a}$

Normal: $y - \frac{3}{2}a^2 = -\frac{1}{3a}(x - a)$

Put x = 0: $y = \frac{3}{2}a^2 + \frac{a}{3a} = \frac{3}{2}a^2 + \frac{1}{3}$

Answer: shown: $y = \frac{3}{2}a^2 + \frac{1}{3}$