Day 185 **IGCSE 10-a-day: Model Solutions**

1 Ordering numbers

Write these numbers in order of size, starting with the smallest.

$\frac{7}{9}$, 0.56, 87%, 0.85

Convert each to a decimal: $\frac{7}{9}$ = 0.778, 0.56 = 0.56, 87% = 0.87, 0.85 = 0.85

In order: 0.56, $\frac{7}{9}$, 0.85, 87%

Answer: 0.56, $\frac{7}{9}$, 0.85, 87%

2 Chords and Pythagoras in circles

AB is a chord of a circle, centre O. M is the midpoint of AB. (Calculator)
(a) Work out the length OM.
(b) Work out angle AOB, to 1 decimal place.
The diagram is not drawn accurately.

(a) AM = 20 ÷ 2 = 10 cm
$OM^2 = OA^2 - AM^2 = 26^2 - 10^2 = 576$
OM = 24 cm

(b) sin(angle AOM) = $\frac{10}{26}$

Angle AOM = 22.62°, so angle AOB = 2 × 22.62 = 45.2°

Answer: (a) 24 cm (b) 45.2°

3 Pressure

A metal crate in the shape of a cuboid weighs 420 N.
Its smallest face has area 200 cm^2. Its largest face has area 800 cm^2.
Pressure = force ÷ area
Work out (a) the greatest pressure and (b) the least pressure the metal crate can exert on the ground, in N/m^2.

1 m^2 = 10 000 cm^2, so 200 cm^2 = 0.02 m^2 and 800 cm^2 = 0.08 m^2

(a) Greatest pressure is on the smallest face: 420 ÷ 0.02 = 21,000 N/m^2

(b) Least pressure is on the largest face: 420 ÷ 0.08 = 5,250 N/m^2

Answer: (a) 21,000 N/m^2 (b) 5,250 N/m^2

4 Fractional power functions

$f(x) = 2x^{1/2} + 4x^{-1/2}$
Find $f'(x)$ and work out the value of $f'(9)$.

$2x^{1/2} \to x^{-1/2}$, $4x^{-1/2} \to -2x^{-3/2}$

$f'(x) = x^{-1/2} - 2x^{-3/2}$

$f'(9)$: $\sqrt{9} = 3$, so $f'(9) = \frac{7}{27}$

Answer: $f'(x) = x^{-1/2} - 2x^{-3/2}$, $f'(9) = \frac{7}{27}$

5 Speed-time graphs

The graph shows the speed of a train between two stations.
(a) Work out the acceleration in the first 10 seconds.
(b) Work out the total distance.

(a) Acceleration = change in speed ÷ time = 30 ÷ 10 = 3 m/s^2

(b) Distance = area under the graph, split into two triangles and a rectangle

$\frac{1}{2} \times 10 \times 30 + 30 \times 8 + \frac{1}{2} \times 30 \times 30 = 150 + 240 + 450$

= 840

Answer: (a) 3 m/s^2 (b) 840 m

6 Solving AX = B

A = $\begin{pmatrix}1 & 1\\4 & 0\end{pmatrix}$ and B = $\begin{pmatrix}5\\24\end{pmatrix}$.

AX = B, where X is a 2 × 1 matrix.

Find X.

$X = A^{-1}B$ (multiply both sides on the left by A^{-1})

$A^{-1} = -\frac{1}{4}\begin{pmatrix}0 & -1\\-4 & 1\end{pmatrix}$

$X = -\frac{1}{4}\begin{pmatrix}0 & -1\\-4 & 1\end{pmatrix}\begin{pmatrix}5\\24\end{pmatrix} = \begin{pmatrix}6\\-1\end{pmatrix}$

Answer: $\mathbf{X} = \begin{pmatrix}\mathbf{6}\\\mathbf{-1}\end{pmatrix}$

7 Gradient where the curve crosses the x-axis

A curve has equation y = (x + 2)(x + 1)(x − 3).

Work out the gradient of the curve at each point where it crosses the x-axis.

Expand: $y = x^3 - 7x - 6$

$\frac{dy}{dx} = 3x^2 - 7$

The curve crosses the x-axis at x = −2, −1 and 3

Gradients: x = −2 gives 5, x = −1 gives −4, x = 3 gives 20

Answer: 5 at x = −2, −4 at x = −1, 20 at x = 3

8 Cumulative frequency graph: median and quartiles

The cumulative frequency graph shows the times, in minutes, taken by 200 runners to finish a race.

(a) Find the median. (b) Find the interquartile range.

(c) How many runners took more than 80?

(a) Median at cumulative frequency 100: reading across gives 80

(b) Lower quartile at 50 gives 40, upper quartile at 150 gives 100

IQR = 100 − 40 = 60

(c) At 80 the cumulative frequency is 100, so 200 − 100 = 100

Answer: (a) 80 (b) 60 (c) 100

9 Ratio and money transferred

Carlos and Beth share some money in the ratio 2 : 7.

Carlos gives Beth £10.

The ratio of their money is now 1 : 5.

How much money do they have altogether?

Let the amounts be 2k and 7k, so the total is 9k

Carlos now has 2k − 10, which is $\frac{1}{6}$ of the total

6(2k − 10) = 1 × 9k

12k − 60 = 9k, so 3k = 60 and k = 20

Total = 9 × 20 = £180

Answer: £180

10 Stationary point of y = ax + b/√x

A curve has equation $y = -2x - \frac{108}{\sqrt{x}}$ for $x > 0$.

Find the coordinates of the stationary point and determine its nature.

$\frac{dy}{dx} = -2 + 54x^{-3/2}$

$\frac{dy}{dx} = 0$: $-54x^{-3/2} = -2$, so $x^{3/2} = 27$

$x = 27^{2/3} = 9$, and $y = -54$

$\frac{d^2y}{dx^2} = -81x^{-5/2}$

At $x = 9$, $\frac{d^2y}{dx^2} = -\frac{1}{3}$, which is negative, so a maximum

Answer: (9, −54), maximum