Day 184 **IGCSE 10-a-day: Model Solutions**

1 Gradient at a point

Work out the gradient of the curve $y = 2x^4 - x^3 + 4x^2$ at the point where $x = -3$.

$\frac{dy}{dx} = 8x^3 - 3x^2 + 8x$

When $x = -3$: gradient $= -267$

Answer: -267

2 Inclusion and exclusion with two sets

There are 49 members of a club.
12 of them play chess and 12 of them play badminton.
27 of them do neither.
(a) Work out how many do both.
(b) One of the members of a club is chosen at random. Find the probability that they only play chess.

Number in A or B $= 49 - 27 = 22$
(a) $n(A \cap B) = n(A) + n(B) - n(A \cup B) = 12 + 12 - 22 = 2$
(b) Number who only play chess $= 12 - 2 = 10$
Probability $= \frac{10}{49}$

Answer: (a) 2 (b) $\frac{10}{49}$

3 Quadratic graphs

The curve $y = x^2 - 8x + 12$ crosses the x-axis at the points A and B.
(a) Find the coordinates of A and B.
(b) Find the coordinates of the minimum point of the curve.

(a) Put $y = 0$: $x^2 - 8x + 12 = 0$, so $(x - 2)(x - 6) = 0$
$x - 2 = 0$ or $x - 6 = 0$, so $x = 2$ or $x = 6$
(b) The turning point is halfway between A and B: $x = (2 + 6) \div 2 = 4$
When $x = 4$: $y = 4^2 - 8 \times 4 + 12 = -4$
Answer: (a) A(2, 0), B(6, 0) (b) (4, -4)

4 Exponential growth and decay

There are 50 bacteria in a sample. The number of bacteria is multiplied by 3 every hour.
After how many hours will there first be more than 5,000 bacteria?

Number after n hours $= 50 \times 3^n$
n = 4: 4,050
n = 5: 12,150
12,150 is the first value above 5,000
Answer: 5 hours

5 HCF and LCM

(a) Write 882 and 2352 as products of prime factors.
(b) Find the HCF and the LCM of 882 and 2352.
Give your answers as numbers.

(a) $882 = 2 \times 3^2 \times 7^2$ and $2352 = 2^4 \times 3 \times 7^2$
(b) HCF $= 2 \times 3 \times 7^2 = 294$
LCM $= 2^4 \times 3^2 \times 7^2 = 7056$
Answer: (a) $2 \times 3^2 \times 7^2$; $2^4 \times 3 \times 7^2$ (b) HCF = 294, LCM = 7056

6 Speed-time graphs

The graph shows the speed of a train between two stations.
(a) Work out the acceleration in the first 5 seconds.
(b) Work out the total distance.

(a) Acceleration = change in speed ÷ time = 10 ÷ 5 = 2 m/s^2
(b) Distance = area under the graph, split into two triangles and a rectangle
$\frac{1}{2} \times 5 \times 10 + 10 \times 15 + \frac{1}{2} \times 5 \times 10 = 25 + 150 + 25$
= 200
Answer: (a) 2 m/s^2 (b) 200 m

7 Show that a line is a tangent

The line $y = x + 8$ meets the curve $y = 3x^2 + x + 8$.
Show that the line is a tangent to the curve and find the coordinates of the point where they touch.

Equate: $3x^2 + x + 8 = x + 8$, so $3x^2 = 0$
Discriminant: $0^2 - 4 \times 3 \times 0 = 0 - 0 = 0$
One repeated root, so the line touches the curve at one point only: it is a tangent
$x = \frac{-0}{2 \times 3} = 0$ and $y = 8$
Answer: (0, 8)

8 Bounds: finding an unknown length

A rectangle has area 87 cm^2, correct to 2 significant figures.
Its length is 13 cm, correct to 2 significant figures.
Work out the lower bound of the width of the rectangle. Give your answer correct to 3 significant figures. (Calculator)

Width = area ÷ length
Area: $86.5 \leq A < 87.5$. Length: $12.5 \leq l < 13.5$
LB width = LB area ÷ UB length = 86.5 ÷ 13.5 = 6.41
Answer: 6.41 cm

9 Show particle never at rest

A particle moves in a straight line. After t seconds its displacement from a fixed point is s metres, where
$s = t^3 - 6t^2 + 17t - 4$
(a) Show that the particle is never at rest.
(b) Find the least speed of the particle and the time it occurs.

$v = \frac{ds}{dt} = 3t^2 - 12t + 17 = 3(t - 2)^2 + 5$
A square is never negative, so $v \geq 5 > 0$ and P is never at rest
Least value of v occurs when t = 2, giving 5
Answer: (b) 5 m/s when t = 2 s

10 Angle in a semicircle

AB is a diameter of a circle, centre O. C is a point on the circle. Angle BAC = 45°.
(a) Work out the size of angle ACB.
(b) Work out the size of angle COB.
The diagram is not drawn accurately.

(a) The angle in a semicircle is 90°, so angle ACB = 90°
(b) OA = OC (radii), so triangle OAC is isosceles and angle OCA = 45°
Angle COB is an exterior angle of triangle OAC: 45 + 45 = 90°
(or: the angle at the centre is twice the angle at the circumference)
Answer: (a) 90° (b) 90°