Day 183 **IGCSE 10-a-day: Model Solutions**

1 Cosine rule: finding a side

Work out the length of BC.
Give your answer correct to 3 significant figures. (Calculator)

$BC^2 = AB^2 + AC^2 - 2 \times AB \times AC \times \cos A$
$BC^2 = 5.1^2 + 12.2^2 - 2 \times 5.1 \times 12.2 \times \cos 139°$
$BC^2 = 268.77$
$BC = 16.4$
Answer: 16.4 cm

2 Product of brackets: tangent gradient

$y = x(x + 3)(2x - 1)$
Work out the gradient of the curve at the point where $x = -1$.

Expand: $y = 2x^3 + 5x^2 - 3x$
$\frac{dy}{dx} = 6x^2 + 10x - 3$
$x = -1$: $\frac{dy}{dx} = -7$
Answer: -7

3 Probability with sets

$n(\xi) = 120$, $n(A) = 49$, $n(B) = 37$ and $n(A \cap B) = 33$.
Find $P(A \mid B)$ and $P(B \mid A')$.

$P(A \mid B) = \frac{n(A \cap B)}{n(B)} = \frac{33}{37}$
$n(A') = 120 - 49 = 71$ and $n(B \cap A') = 37 - 33 = 4$
$P(B \mid A') = \frac{4}{71}$
Answer: $P(A \mid B) = \frac{33}{37}$, $P(B \mid A') = \frac{4}{71}$

4 Show that dy/dx = ...

$y = \frac{(x + 3)^2}{x}$ $(x \neq 0)$
(a) Show that $\frac{dy}{dx} = \frac{(x - 3)(x + 3)}{x^2}$.
(b) Find the gradient when $x = 4$.

Expand: $(x + 3)^2 = x^2 + 6x + 9$, so $y = x + 6 + 9x^{-1}$
$\frac{dy}{dx} = 1 - \frac{9}{x^2} = \frac{x^2 - 9}{x^2} = \frac{(x - 3)(x + 3)}{x^2}$
When $x = 4$: $\frac{dy}{dx} = \frac{7}{16}$
Answer: (b) $\frac{7}{16}$

5 Compound interest

Dina puts £10,000 in a savings account.
Compound interest is paid at 6% in year 1, 3% in year 2 and 4% in year 3.
Work out the amount in the account at the end of year 3.
Give your answer to the nearest penny. (Calculator)

Multipliers: 1.06, 1.03, 1.04
£10,000 × 1.06 × 1.03 × 1.04
= £11,354.72
Answer: £11,354.72

6 Gradient where the curve crosses the x-axis

A curve has equation $y = (x + 4)(x + 3)(x - 1)$.
Work out the gradient of the curve at each point where it crosses the x-axis.

Expand: $y = x^3 + 6x^2 + 5x - 12$

$\frac{dy}{dx} = 3x^2 + 12x + 5$

The curve crosses the x-axis at $x = -4$, -3 and 1
Gradients: $x = -4$ gives 5, $x = -3$ gives -4, $x = 1$ gives 20
Answer: 5 at $x = -4$, -4 at $x = -3$, 20 at $x = 1$

7 Terms of a quadratic sequence

The nth term of a sequence is $n^2 + n + 4$.
(a) Work out the 11th term.
(b) Is 346 a term of the sequence? You must show your working.

(a) $11^2 + 1 \times 11 + 4 = 136$
(b) Solve $n^2 + n + 4 = 346$, so $n^2 + n - 342 = 0$
Factorise or use the formula: $n = 18$ or $n = -19$
$n = 18$ is a positive whole number, so 346 is term number 18
Answer: (a) 136 (b) Yes, it is the 18th term

8 Speed and a moving train

A train travels at a constant speed of 90 km/h.
It takes 60 seconds to cross completely over a bridge which is 1,310 m long.
Work out the length of the train, in metres.

Speed = 90 km/h = 90,000 ÷ 3600 = 25 m/s
Distance travelled in 60 s = 25 × 60 = 1,500 m
This is the length of the bridge plus the length of the train
Train = 1,500 − 1,310 = 190 m
Answer: 190 m

9 Equations in disguise

Solve $x^4 - 65x^2 + 784 = 0$

Let $u = x^2$: $u^2 - 65u + 784 = 0$
Factorise: $(u - 49)(u - 16) = 0$, so $u = 49$ or $u = 16$
$x^2 = 49$ or $x^2 = 16$
$x = \pm 7$ or $x = \pm 4$
Answer: $x = -7, -4, 4, 7$

10 Perpendicular bisector

A is the point $(-1, 0)$ and B is the point $(-3, -8)$.
Find an equation of the perpendicular bisector of AB. Give your answer in the form $ax + by = c$, where a, b and c are integers.

Midpoint of AB = $(-2, -4)$
Gradient of AB = 4

Perpendicular gradient = $-\frac{1}{4}$

$y - (-4) = (-\frac{1}{4})(x - (-2))$

Rearrange to integer coefficients: $x + 4y = -18$
Answer: $x + 4y = -18$