Day 182 **IGCSE 10-a-day: Model Solutions**

1 Adding and scaling matrices

$A = \begin{pmatrix} 3 & -3 \\ 5 & -2 \end{pmatrix}$ and $B = \begin{pmatrix} 5 & 3 \\ 4 & -2 \end{pmatrix}$.

(a) Work out A + B.

(b) Work out 2A − 2B.

(a) $A + B = \begin{pmatrix} 8 & 0 \\ 9 & -4 \end{pmatrix}$

(b) $2A = \begin{pmatrix} 6 & -6 \\ 10 & -4 \end{pmatrix}$ and $2B = \begin{pmatrix} 10 & 6 \\ 8 & -4 \end{pmatrix}$

$2A - 2B = \begin{pmatrix} -4 & -12 \\ 2 & 0 \end{pmatrix}$

Answer: (a) $\begin{pmatrix} \mathbf{8} & \mathbf{0} \\ \mathbf{9} & \mathbf{-4} \end{pmatrix}$ **(b)** $\begin{pmatrix} \mathbf{-4} & \mathbf{-12} \\ \mathbf{2} & \mathbf{0} \end{pmatrix}$

2 Ordering numbers

Write these numbers in order of size, starting with the smallest.

0.41, $\frac{1}{4}$, $\frac{13}{18}$, 85%

Convert each to a decimal: 0.41 = 0.41, $\frac{1}{4}$ = 0.25, $\frac{13}{18}$ = 0.722, 85% = 0.85

In order: $\frac{1}{4}$, 0.41, $\frac{13}{18}$, 85%

Answer: $\frac{1}{4}$, 0.41, $\frac{13}{18}$, 85%

3 Factorising fully

Factorise fully $2x^3 - 32x$

Take out the common factor 2x: $2x(x^2 - 16)$

$x^2 - 16$ is a difference of two squares: $(x - 4)(x + 4)$

Answer: $2x(x - 4)(x + 4)$

4 Overall percentage change from repeated changes

The price of a coat is decreased by 50% and then increased by 20%.

Work out the overall percentage change. State whether it is an increase or a decrease.

Multipliers: 0.5 and 1.2

Overall multiplier = 0.5 × 1.2 = 0.6

40% decrease

Answer: 40% decrease

5 Where a line meets the axes

The line L has equation 3x + 4y = 12.

L crosses the x-axis at the point A and the y-axis at the point B.

(a) Find the coordinates of A and B.

(b) Work out the length of AB.

(a) At A, y = 0: 3x = 12, so x = 4. A is (4, 0)

At B, x = 0: 4y = 12, so y = 3. B is (0, 3)

(b) $AB^2 = 4^2 + 3^2 = 25$

$AB = \sqrt{25} = 5$

Answer: (a) A(4, 0), B(0, 3) (b) 5

6 Nature from the gradient either side

$y = 2x^3 + 3x^2 - 12x - 3$

(a) Show that there is a stationary point at $x = -2$.

(b) By finding the gradient when $x = -\frac{5}{2}$ and when $x = -\frac{3}{2}$, decide whether it is a maximum or a minimum.

$\frac{dy}{dx} = 6x^2 + 6x - 12$, so when $x = -2$: $\frac{dy}{dx} = 0$

$x = -\frac{5}{2}$: gradient $= \frac{21}{2}$ (positive)

$x = -\frac{3}{2}$: gradient $= -\frac{15}{2}$ (negative)

The gradient goes from positive to 0 to negative, so $(-2, 17)$ is a maximum

Answer: (b) maximum

7 Arc length and sector area

A sector of a circle has angle 270° and area $\frac{675}{4}\pi$ cm^2.

(a) Work out the radius of the circle.

(b) Work out the length of the arc of the sector. Give your answer to (b) in terms of π.

(a) Area $= \frac{270}{360} \times \pi \times r^2$, so $\frac{270}{360} \times r^2 = \frac{675}{4}$

$r^2 = \frac{675}{4} \times \frac{360}{270} = 225$

$r = 15$ cm

(b) Arc length $= \frac{270}{360} \times 2 \times \pi \times 15 = \frac{45}{2}\pi$ cm

Answer: (a) 15 cm (b) $\frac{45}{2}\pi$ cm

8 Show particle never at rest

A particle moves in a straight line. After t seconds its displacement from a fixed point is s metres, where $s = 2t^3 - 18t^2 + 61t - 4$

(a) Show that the particle is never at rest.

(b) Find the least speed of the particle and the time it occurs.

$v = \frac{ds}{dt} = 6t^2 - 36t + 61 = 6(t - 3)^2 + 7$

A square is never negative, so $v \geq 7 > 0$ and P is never at rest

Least value of v occurs when $t = 3$, giving 7

Answer: (b) 7 m/s when t = 3 s

9 Histograms with an unmarked scale

The histogram shows the times of people spent waiting at a clinic.

320 of them had a time between 20 and 40 minutes.

(a) How many had a time between 40 and 60 minutes?

(b) How many people are there in total?

Frequency is proportional to the area of a bar

20 to 40: bar is 4 squares high and 20 wide, area 80, which represents 320

So 1 unit of area represents $320 \div 80 = 4$

(a) 40 to 60: $5 \times 20 = 100$ units of area, so $100 \times 4 = 400$

(b) Total area $= 60 + 70 + 80 + 100 + 40 = 350$, so total $= 350 \times 4 = 1400$

Answer: (a) 400 (b) 1400

10 Show that dy/dx factorises

$y = x(x + 2)^2$

Show that $\frac{dy}{dx} = (x + 2)(3x + 2)$.

Expand: $y = x^3 + 4x^2 + 4x$

$\frac{dy}{dx} = 3x^2 + 8x + 4$

Factorise: $3x^2 + 8x + 4 = (x + 2)(3x + 2)$

Answer: $\mathbf{\frac{dy}{dx} = (x + 2)(3x + 2)}$