Day 181 **IGCSE 10-a-day: Model Solutions**

1 Integers satisfying a double inequality

n is an integer.
Work out the sum of all the values of n that satisfy
$2 < 3n - 5 < 15$

Add 5 from each part: $7 < 3n < 20$
Divide each part by 3: $\frac{7}{3} < n < \frac{20}{3}$
Integers: 3, 4, 5, 6
Sum = 18
Answer: 18

2 Two numbers: maximise or minimise

The positive numbers x and y satisfy $x + y = 18$.
Find the greatest possible value of x^2y, showing that it is the greatest.

$y = 18 - x$, so $x^2y = -x^3 + 18x^2$
$\frac{d}{dx}(x^2y) = -3x^2 + 36x = 0$, so $x(36 - 3x) = 0$ and $x = 12$ $(x > 0)$
Second derivative $= -6x + 36 = -36 < 0$, so a maximum
$y = 6$, so the greatest value is $12^2 \times 6 = 864$
Answer: 864

3 Bounds in geometry

A rectangle has length 5.7 cm and width 6.4 cm, both correct to 1 decimal place.
Work out the lower bound of the perimeter of the rectangle.

Each length is given to 1 decimal place, so it can be up to 0.05 cm either side
Length is between 5.65 and 5.75, width is between 6.35 and 6.45
Perimeter $= 2 \times (5.65 + 6.35) = 24$ cm
Answer: 24 cm

4 Rearrange then differentiate

$y = \frac{(4x + 2)^2}{x}$

Find $\frac{dy}{dx}$.

Expand the bracket: $y = \frac{16x^2 + 16x + 4}{x} = 16x + 16 + 4x^{-1}$
$16x \to 16$, $4x^{-1} \to -4x^{-2}$
$\frac{dy}{dx} = 16 - 4x^{-2}$
Answer: $\mathbf{\frac{dy}{dx} = 16 - \frac{4}{x^2}}$

5 Exact trigonometry in triangles

ABC is a right-angled triangle.
Work out the exact value of x. Give your answer in its simplest form.
The diagram is not drawn accurately.

x is the hypotenuse AC and the known side is the adjacent side AB, so use cos 60°
$\cos 60° = \frac{1}{2}$
$x = 12 \div \cos 60° = 12 \div \frac{1}{2}$
$x = 24$
Answer: x = 24

6 Equations with algebraic fractions

Solve $\frac{1}{x-5} + \frac{5}{x+2} = \frac{1}{2}$

Multiply every term by $(x-5)(x+2)$: $(x+2) + 5(x-5) = \frac{1}{2} \times (x-5)(x+2)$

Multiply by 2 to clear the fraction: $2(x+2) + 10(x-5) = (x-5)(x+2)$

Expand and rearrange so that one side is 0: $x^2 - 15x + 36 = 0$

Factorise: $(x-12)(x-3) = 0$

$x = 3$ or $x = 12$

Answer: $x = 3$ or $x = 12$

7 Percentages of percentages

In a group of students, 40% are boys and the rest are girls.
60% of the boys and 20% of the girls walk to school.
What percentage of all the students walk to school?

Take 100 students: 40 are boys and 60 are girls

60% of 40 = 24 and 20% of 60 = 12

24 + 12 = 36 out of 100

Answer: 36%

8 Reverse percentage growth and decay

The value of a house increases by 25% each year.
After 2 years it is worth £13,281.25.
Work out its value 2 years ago. (Calculator)

Yearly multiplier = 1.25

Original $\times 1.25^2$ = £13,281.25

Original = £13,281.25 $\div 1.25^2$ = £8,500

Answer: £8,500

9 Vector of given length parallel to another

$\mathbf{p} = \begin{pmatrix} 8 \\ 15 \end{pmatrix}$.

Find a vector of length 51 that is parallel to $\mathbf{p}$.

$|\mathbf{p}| = \sqrt{(8)^2 + (15)^2} = \sqrt{289} = 17$

Scale factor = 51 ÷ 17 = 3

$3\mathbf{p} = \begin{pmatrix} 24 \\ 45 \end{pmatrix}$

Answer: $\begin{pmatrix} 24 \\ 45 \end{pmatrix}$ (or $\begin{pmatrix} -24 \\ -45 \end{pmatrix}$)

10 Stationary points of a quartic

A curve has equation $y = 2x^4 - 36x^2 + 3$.
Find the coordinates of the stationary points of the curve and determine the nature of each.

$\frac{dy}{dx} = 8x^3 - 72x = 8x(x-3)(x+3)$

$\frac{dy}{dx} = 0$ gives $x = 0$, $x = 3$ or $x = -3$

$y = 3$ when $x = 0$, and $y = -159$ when $x = \pm 3$

$\frac{d^2y}{dx^2} = 24x^2 - 72$: it is -72 at $x = 0$ (maximum) and 144 at $x = \pm 3$ (minimum)

Answer: (0, 3) maximum; (3, −159) and (−3, −159) minima