Day 180 **IGCSE 10-a-day: Model Solutions**

1 Angles in polygons

The interior angles of a pentagon are
$x°$, $(x + 5)°$, $(x - 5)°$, $(x + 10)°$, $(x + 30)°$
(a) Work out the value of x.
(b) Work out the size of the largest angle.

(a) Sum of interior angles = $(5 - 2) \times 180 = 540°$
$5x + 40 = 540$, so $5x = 500$
$x = 100$
(b) The angles are 100°, 105°, 95°, 110°, 130°
The largest is 130°
Answer: (a) x = 100 (b) 130°

2 Fractions removed from a ratio

The ratio of blue to green sweets in a tin is 3 : 8.
Then $\frac{1}{3}$ of the blue and $\frac{1}{2}$ of the green are taken out.
Work out the ratio of blue to green now.

Left: $\frac{2}{3} \times 3 = 2$ of the blue part, $\frac{1}{2} \times 8 = 4$ of the green part
2 : 4
Multiply by 1: 2 : 4 = 1 : 2
Answer: 1 : 2

3 Nature from the gradient either side

$y = -2x^3 + 6x - 4$
(a) Show that there is a stationary point at x = 1.
(b) By finding the gradient when $x = \frac{1}{2}$ and when $x = \frac{3}{2}$, decide whether it is a maximum or a minimum.

$\frac{dy}{dx} = -6x^2 + 6$, so when x = 1: $\frac{dy}{dx} = 0$
$x = \frac{1}{2}$: gradient = $\frac{9}{2}$ (positive)
$x = \frac{3}{2}$: gradient = $-\frac{15}{2}$ (negative)
The gradient goes from positive to 0 to negative, so (1, 0) is a maximum
Answer: (b) maximum

4 Fraction problems

A bag contains marbles.
$\frac{1}{4}$ of the marbles are yellow and $\frac{1}{5}$ of the marbles are black.
The rest of the marbles, 11 of them, are white.
How many black marbles are there?

Fraction that are white = $1 - \frac{1}{4} - \frac{1}{5} = \frac{11}{20}$
$\frac{11}{20}$ of the total is 11
Total = $11 \div \frac{11}{20} = 20$
Black marbles = $\frac{1}{5}$ of 20 = 4
Answer: 4

5 Fractional power functions

$f(x) = -6x^{3/2} - 2x - 6x^{-1/2}$

Find $f'(x)$ and work out the value of $f'(4)$.

$-6x^{3/2} \to -9x^{1/2}$, $-2x \to -2$, $-6x^{-1/2} \to 3x^{-3/2}$

$f'(x) = -9x^{1/2} - 2 + 3x^{-3/2}$

$f'(4)$: $\sqrt{4} = 2$, so $f'(4) = -\frac{157}{8}$

Answer: $f'(x) = -9x^{1/2} - 2 + 3x^{-3/2}$, $f'(4) = -\frac{157}{8}$

6 Composite functions

$f(x) = 3x + 3$ and $g(x) = 5x^2 + 3$

(a) Find $fg(x)$.

(b) Solve $fg(x) = 72$

(a) $fg(x) = f(5x^2 + 3) = 3(5x^2 + 3) + 3$

$= 15x^2 + 12$

(b) $15x^2 + 12 = 72$, so $15x^2 = 60$

$x^2 = 4$, so $x = \pm 2$

Answer: (a) $15x^2 + 12$ (b) $x = \pm 2$

7 Tangent through a given point: find the constant

A curve has equation $y = -x^2 - 6x + k$, where k is a constant.

The tangent to the curve at the point where $x = 3$ passes through $(4, -38)$.

Find the value of k.

$\frac{dy}{dx} = -2x - 6$, so at $x = 3$ the gradient is -12

The point of tangency is $(3, -27 + k)$

Gradient between this point and $(4, -38)$: $\frac{-38 - (-27 + k)}{4 - 3} = -12$

$-38 - (-27 + k) = -12$

$k = 1$

Answer: $k = 1$

8 Show that: consecutive terms of a quadratic sequence

The nth term of a sequence is $4n^2 + 7n + 8$.

(a) Show that the difference between the $(n + 1)$th term and the nth term is $8n + 11$.

(b) Two consecutive terms of the sequence differ by 155. Find the value of n for the first of these two terms.

(a) $(n + 1)$th term $= 4(n + 1)^2 + 7(n + 1) + 8 = 4n^2 + 15n + 19$

Subtract the nth term: $4n^2 + 15n + 19 - (4n^2 + 7n + 8) = 8n + 11$

(b) $8n + 11 = 155$ so $8n = 144$

$n = 18$

Answer: (b) $n = 18$

9 Functions that are their own inverse

$f(x) = \frac{2x - 1}{2x - 2}$ where $x \neq 1$

(a) Show that $ff(x) = x$.

(b) Write down $f^{-1}(x)$.

(a) Replace x in f(x) by f(x), then multiply the top and the bottom by $(2x - 2)$

$ff(x) = \frac{2(2x - 1) + (-1)(2x - 2)}{2(2x - 1) - 2(2x - 2)}$

Top $= 4x - 2 - 2x + 2 = 2x$

Bottom $= 4x - 2 - 4x + 4 = 2$

$ff(x) = \frac{2x}{2} = x$

(b) $ff(x) = x$, so f is its own inverse: $f^{-1}(x) = \frac{2x - 1}{2x - 2}$

Answer: (b) $\mathbf{f^{-1}(x) = \frac{2x - 1}{2x - 2}}$

10 Show that lines are parallel

In triangle OAB, $\mathbf{OA} = \mathbf{a}$ and $\mathbf{OB} = \mathbf{b}$.

M is on OA and N is on OB, with OM : MA = ON : NB = 2 : 1.

(a) Show that MN is parallel to AB.

(b) Write down the ratio MN : AB.

(a) $\mathbf{OM} = \frac{2}{3}\mathbf{a}$ and $\mathbf{ON} = \frac{2}{3}\mathbf{b}$

$\mathbf{MN} = \mathbf{MO} + \mathbf{ON} = -\frac{2}{3}\mathbf{a} + \frac{2}{3}\mathbf{b} = \frac{2}{3}(\mathbf{b} - \mathbf{a})$

$\mathbf{b} - \mathbf{a} = \mathbf{AB}$, so $\mathbf{MN} = \frac{2}{3}\mathbf{AB}$: a multiple of $\mathbf{AB}$, so MN is parallel to AB

(b) MN : AB = 2 : 3

Answer: (b) 2 : 3