Day 18 **IGCSE 10-a-day: Model Solutions**

1 Parallelogram and midpoint with vectors

ABCD is a parallelogram.
A = (4, 3), B = (0, 0) and C = (5, −4).
Find the coordinates of D.

AD = **BC** because opposite sides are equal and parallel: $\begin{pmatrix} 5 \\ -4 \end{pmatrix}$

D = A + **BC** = (4, 3) + $\begin{pmatrix} 5 \\ -4 \end{pmatrix}$

D = (9, −1)

Answer: (9, −1)

2 Coordinates and gradient at the y-intercept

$y = -2x^3 - 5x^2 + 5x - 4$
The curve crosses the y-axis at the point A.
Work out the coordinates of A and the gradient of the curve at A.

When x = 0, y = −4, so A is (0, −4)

$\frac{dy}{dx} = -6x^2 - 10x + 5$

When x = 0, $\frac{dy}{dx} = 5$

Answer: A = (0, −4), gradient 5

3 Magnitude of a vector

$\mathbf{v} = \begin{pmatrix} k \\ 8 \end{pmatrix}$ and $|\mathbf{v}| = 17$.
Find the two possible values of k.

$k^2 + 64 = 289$, so $k^2 = 225$
k = 15 or k = −15

Answer: k = 15 or k = −15

4 No stationary points

A curve has equation $y = x^3 + 3x^2 + 8x$.
(a) Show that the curve has no stationary points.
(b) Find the least value of the gradient of the curve.

$\frac{dy}{dx} = 3x^2 + 6x + 8 = 3(x + 1)^2 + 5$

A squared term is never negative, so $\frac{dy}{dx} \geq 5 > 0$ for all x

So $\frac{dy}{dx}$ is never zero, and there are no stationary points

The least gradient is 5 (when x = −1)

Answer: (b) 5

5 Percentage change

A price is decreased by 36%.
By what percentage must the new value be increased to get back to the original value?

Let the original be 100. After a 36% decrease it is 64.
To return to 100, increase by 36

Percentage increase = $\frac{36}{64} \times 100 = 56.25\%$

Answer: 56.25%

6 Equation of a curve from a sketch

The diagram shows a sketch of a quadratic curve. It crosses the axes at the points shown.
Find an equation of the curve.
Give your answer in the form $y = ax^2 + bx + c$.

The roots are $x = -3$ and $x = 2$, so $y = a(x + 3)(x - 2)$
When $x = 0$, $y = 6$: $a \times 3 \times (-2) = 6$, so $a = -1$
$y = -(x + 3)(x - 2)$
$y = -x^2 - x + 6$
Answer: $y = -x^2 - x + 6$

7 Rates of work

Anna can paint a fence in 12 hours.
Ben can paint the same fence in 60 hours.
They work together at these rates.
How long do they take to paint the fence?

In 1 hour: Anna does $\frac{1}{12}$ and Ben does $\frac{1}{60}$
Together: $\frac{1}{12} + \frac{1}{60} = \frac{72}{720}$
Time = $\frac{720}{72}$ = 10 hours
Answer: 10 hours

8 Tangent perpendicular to a line

The tangent at the point P on the curve $y = 2x^2 + x - 8$ is perpendicular to the line $x + 9y = 2$.
Find the coordinates of P.

The line is $y = -\frac{1}{9}x + \frac{2}{9}$, so its gradient is $-\frac{1}{9}$
The tangent has gradient 9
$\frac{dy}{dx} = 4x + 1 = 9$ so $x = 2$
$y = 2$
Answer: (2, 2)

9 Fractional indices

Given that $x > 0$, solve $x^{-\frac{5}{3}} = \frac{1}{32}$

Raise both sides to the power $-\frac{3}{5}$
$x = (\frac{1}{32})^{-\frac{3}{5}}$
$x = 8$
Answer: $x = 8$

10 Pythagoras with algebra

ABC is a right-angled triangle with sides $(x - 3)$ cm, $(2x + 4)$ cm and $(2x + 5)$ cm. The longest side is $(2x + 5)$ cm.
(a) Find the value of x.
(b) Work out the area of the triangle.
The diagram is not drawn accurately.

(a) Pythagoras: $(x - 3)^2 + (2x + 4)^2 = (2x + 5)^2$
Expand and collect terms: $x^2 - 10x = 0$
Factorise: $x(x - 10) = 0$
$x = 10$ or $x = 0$, and x must be positive, so $x = 10$
(b) The two shorter sides are 7 cm and 24 cm

Area $= \frac{1}{2} \times 7 \times 24 = 84$ cm^2

Answer: (a) x = 10 (b) 84 cm^2