Day 179 **IGCSE 10-a-day: Model Solutions**

1 Multiplying and dividing surds

Simplify fully $3\sqrt{20} \times 2\sqrt{12}$

$3\sqrt{20} \times 2\sqrt{12} = 6\sqrt{240}$

$\sqrt{240} = 4\sqrt{15}$, so $6 \times 4\sqrt{15} = 24\sqrt{15}$

Answer: $24\sqrt{15}$

2 Negative powers

Differentiate $-3x^2 + 4x + \frac{2}{x}$ with respect to x.

Write negative powers as indices: $y = -3x^2 + 4x + 2x^{-1}$

$-3x^2 \rightarrow -6x$, $4x \rightarrow 4$, $2x^{-1} \rightarrow -2x^{-2}$

$\frac{dy}{dx} = -6x + 4 - 2x^{-2}$

Answer: $\frac{dy}{dx} = -6x + 4 - \frac{2}{x^2}$

3 Depreciation

The value of a machine falls by 10% each year.
After 3 years it is worth £3,645.
Work out its value at the start. (Calculator)

Multiplier = 0.9, so value after 3 years = original $\times 0.9^3$

$0.9^3 = 0.729$

Original = $3645 \div 0.729$

= £5,000

Answer: £5,000

4 Pressure

A wooden block in the shape of a cuboid weighs 900 N.
Its smallest face has area 200 cm^2. Its largest face has area 250 cm^2.
Pressure = force ÷ area
Work out (a) the greatest pressure and (b) the least pressure the wooden block can exert on the ground, in N/m^2.

1 m^2 = 10 000 cm^2, so 200 cm^2 = 0.02 m^2 and 250 cm^2 = 0.025 m^2

(a) Greatest pressure is on the smallest face: $900 \div 0.02 = 45{,}000$ N/m^2

(b) Least pressure is on the largest face: $900 \div 0.025 = 36{,}000$ N/m^2

Answer: (a) 45,000 N/m^2 (b) 36,000 N/m^2

5 Area of a triangle using sine

In triangle PQR, PQ = 19 cm and angle PQR = 60°.

The area of the triangle is $\frac{133\sqrt{3}}{2}$ cm^2.

Work out the length of QR.
Do not use a calculator.

Area = $\frac{1}{2} \times PQ \times QR \times \sin Q$

$\frac{133\sqrt{3}}{2} = \frac{1}{2} \times 19 \times QR \times \sin 60°$

$\sin 60° = \frac{\sqrt{3}}{2}$, so $\frac{133\sqrt{3}}{2} = \frac{19\sqrt{3}}{4} \times QR$

$QR = \frac{133\sqrt{3}}{2} \div \frac{19\sqrt{3}}{4} = 14$ cm

Answer: 14 cm

6 Compound interest: how many years

£2,500 is invested at 10% per year compound interest.
Work out the least whole number of years after which the investment is worth more than £7,500. (Calculator)

Value after n years = 2500×1.1^n
After 11 years: £7,132.79 (less than £7,500)
After 12 years: £7,846.07 (more than £7,500)
Answer: 12 years

7 Laws of indices with algebra

Simplify $(25x^4y^4)^{\frac{3}{2}}$

Raise each part to the power $\frac{3}{2}$

$25^{\frac{3}{2}} = (\sqrt{25})^3 = 5^3 = 125$

$(x^4)^{\frac{3}{2}} = x^6$ and $(y^4)^{\frac{3}{2}} = y^6$

$= 125x^6y^6$

Answer: $125x^6y^6$

8 Show particle never at rest

A particle moves in a straight line. After t seconds its displacement from a fixed point is s metres, where
$s = 2t^3 - 18t^2 + 55t - 1$
(a) Show that the particle is never at rest.
(b) Find the least speed of the particle and the time it occurs.

$v = \frac{ds}{dt} = 6t^2 - 36t + 55 = 6(t - 3)^2 + 1$

A square is never negative, so $v \geq 1 > 0$ and P is never at rest
Least value of v occurs when t = 3, giving 1
Answer: (b) 1 m/s when t = 3 s

9 Functions that are their own inverse

$f(x) = \frac{-x + 3}{2x + 1}$ where $x \neq -\frac{1}{2}$
(a) Show that ff(x) = x.
(b) Write down $f^{-1}(x)$.

(a) Replace x in f(x) by f(x), then multiply the top and the bottom by (2x + 1)

$ff(x) = \frac{(-1)(-x + 3) + 3(2x + 1)}{2(-x + 3) - (-1)(2x + 1)}$

Top = $x - 3 + 6x + 3 = 7x$
Bottom = $-2x + 6 + 2x + 1 = 7$

$ff(x) = \frac{7x}{7} = x$

(b) ff(x) = x, so f is its own inverse: $f^{-1}(x) = \frac{-x + 3}{2x + 1}$

Answer: (b) $\mathbf{f^{-1}(x) = \frac{-x + 3}{2x + 1}}$

10 Minimum surface area of a tank

A storage unit is a cuboid with a square base of side x metres and height h metres, and has an open top. Its volume is 32 m^3.

(a) Show that the total outside surface area, A m^2, is $A = x^2 + \frac{128}{x}$.

(b) Find the minimum possible value of A, showing that it is a minimum.

$V = x^2h = 32$, so $h = \frac{32}{x^2}$

$A = x^2 + 4xh = x^2 + 4x \times \frac{32}{x^2} = x^2 + \frac{128}{x}$

$\frac{dA}{dx} = 2x - 128x^{-2} = 0$ gives $x^3 = 64$, so $x = 4$

$\frac{d^2A}{dx^2} = 2 + 256x^{-3} = 6 > 0$, so a minimum

$A = 48$

Answer: 48 m^2