Day 178 **IGCSE 10-a-day: Model Solutions**

1 Equations with fractions

Solve $\frac{x-3}{2} - \frac{x-1}{8} = 2$

Multiply every term by 8: $4(x-3) - (x-1) = 16$
Expand: $3x - 11 = 16$
$3x = 27$
$x = 9$
Answer: $x = 9$

2 Function values and composites

$f(x) = -x + 3$ and $g(x) = 2x^2 + x + 6$.
(a) Work out g(1).
(b) Work out fg(3).

(a) $g(1) = 9$
(b) $g(3) = 27$, so $fg(3) = f(27) = -24$
Answer: (a) 9 (b) −24

3 Turning point of a quadratic by differentiation

$f(x) = x^2 - 8x + 7$
Use differentiation to find the coordinates of the turning point of the graph of $y = f(x)$. State whether it is a maximum or a minimum.

$f'(x) = 2x - 8$
$f'(x) = 0$: $2x - 8 = 0$, so $x = 4$
$f(4) = -9$
$f''(x) = 2$, which is positive, so it is a minimum
Answer: (4, −9), minimum

4 Venn diagram with three sets: reading

The Venn diagram shows information about 40 students.
A: study Art B: study Music C: study Drama
(a) Find $n(B \cup C)'$.
(b) Find $n((A \cap B) \cup C)$.

(a) $n(B \cup C)' = 4 + 3 = 7$
(b) $n((A \cap B) \cup C) = 5 + 7 + 8 + 1 + 2 = 23$
Answer: (a) 7 (b) 23

5 Alternate segment theorem

PAQ is a tangent to the circle at A. B and C are points on the circle. Angle PAB = 40° and angle CAQ = 61°.
(a) Work out the size of angle ABC.
(b) Work out the size of angle ACB.
The diagram is not drawn accurately.

(a) Angle ABC = angle CAQ = 61° (alternate segment theorem)
(b) Angle ACB = angle PAB = 40° (alternate segment theorem)
Answer: (a) 61° (b) 40°

6 Tangent to a factorised cubic

A curve has equation $y = (x-2)(x-2)(x-2)$.
Find the equation of the tangent to the curve at the point where $x = -2$.

Expand: $y = x^3 - 6x^2 + 12x - 8$
$\frac{dy}{dx} = 3x^2 - 12x + 12$
At $x = -2$: $y = -64$ and gradient = 48
$y + 64 = 48(x + 2)$
$y = 48x + 32$
Answer: $y = 48x + 32$

7 Percentages of percentages

In a group of students, 65% are boys and the rest are girls.
30% of the boys and 35% of the girls walk to school.
What percentage of all the students walk to school?

Take 100 students: 65 are boys and 35 are girls
30% of 65 = 19.5 and 35% of 35 = 12.25
19.5 + 12.25 = 31.75 out of 100
Answer: 31.75%

8 Show that a line is a tangent

The line $y = 3x + k$ is a tangent to the curve $y = -2x^2 - 5x - 4$.
Find the value of the constant k.

$\frac{dy}{dx} = -4x - 5 = 3$, so $x = -2$
$y = -2$ when $x = -2$
The line passes through $(-2, -2)$: $-2 = 3 \times (-2) + k$, so $k = 4$
Answer: $k = 4$

9 Density of an alloy

An alloy contains 355 g of zinc and 395 g of iron.
The density of zinc is 7.1 g/cm^3. The density of iron is 7.9 g/cm^3.
Work out the density of the alloy.

Volume of zinc = 355 ÷ 7.1 = 50 cm^3
Volume of iron = 395 ÷ 7.9 = 50 cm^3
Total mass = 750 g, total volume = 100 cm^3
Density = 750 ÷ 100 = 7.5 g/cm^3
Answer: 7.5 g/cm^3

10 Bounds and accuracy

a = 46.4 and b = 0.692, both correct to 3 significant figures.
Work out a ÷ b.
Write down your answer to a suitable degree of accuracy.
You must show your working and give a reason for your answer. (Calculator)

$46.35 \leq a < 46.45$; $0.6915 \leq b < 0.6925$
Lower bound = 46.35 ÷ 0.6925 = 66.931...
Upper bound = 46.45 ÷ 0.6915 = 67.173...
Both bounds round to 67 to 2 significant figures, but not to 3 significant figures
Answer: 67 (2 s.f.)