Day 177 **IGCSE 10-a-day: Model Solutions**

1 Increasing and decreasing intervals

$y = -2x^3 + 6x^2 - 8$

Find the range of values of x for which y is decreasing.

$\frac{dy}{dx} = -6x^2 + 12x = -6x(x - 2)$

$\frac{dy}{dx} = 0$ when $x = 0$ and $x = 2$

The graph of $\frac{dy}{dx}$ against x is a ∩-shaped quadratic

y is increasing when $\frac{dy}{dx} > 0$ and decreasing when $\frac{dy}{dx} < 0$

So y is decreasing when $x < 0$ or $x > 2$

Answer: $x < 0$ or $x > 2$

2 Simplifying surds

Write $\sqrt{252} + \sqrt{112} + \sqrt{63}$ in the form $a\sqrt{7}$

where a is an integer.

$\sqrt{252} = 6\sqrt{7}$; $\sqrt{112} = 4\sqrt{7}$; $\sqrt{63} = 3\sqrt{7}$

$(6 + 4 + 3)\sqrt{7} = 13\sqrt{7}$

Answer: $13\sqrt{7}$

3 Averages with algebra

Here are five numbers: $2x - 1$, x, $2x + 2$, $x - 1$, $x + 3$.

The mean of the five numbers is 9.

(a) Work out the value of x.

(b) Work out the range of the five numbers.

(a) Total $= (2x - 1) + (x) + (2x + 2) + (x - 1) + (x + 3) = 9 \times 5 = 45$

$7x + 3 = 45$, so $7x = 42$ and $x = 6$

The numbers are 11, 6, 14, 5, 9

(b) Range $= 14 - 5 = 9$

Answer: (a) $x = 6$ (b) 9

4 Stationary points and their nature

$y = x^3 - 9x^2 + 24x + 9$

(a) Find the coordinates of the stationary points.

(b) Use $\frac{d^2y}{dx^2}$ to decide whether each is a maximum or a minimum.

$\frac{dy}{dx} = 3x^2 - 18x + 24 = 3(x - 2)(x - 4)$

$\frac{dy}{dx} = 0$ gives $x = 2$ or $x = 4$, so the points are (2, 29) and (4, 25)

$\frac{d^2y}{dx^2} = 6x - 18$

$x = 2$: $\frac{d^2y}{dx^2} = -6$, which is negative, so a maximum

$x = 4$: $\frac{d^2y}{dx^2} = 6$, which is positive, so a minimum

Answer: (a) (2, 29) and (4, 25) (b) (2, 29) is a maximum, (4, 25) is a minimum

5 Powers of standard form

Work out the square root of 2.56×10^{-8}.
Give your answer in standard form.

$2.56 = 1.6 \times 1.6$
$\sqrt{(10^{-8})} = 10^{-4}$
$\sqrt{(2.56 \times 10^{-8})} = 1.6 \times 10^{-4}$
Answer: 1.6×10^{-4}

6 Hollow cylinder and density

A metal pipe is a hollow cylinder of length 30 cm. (Calculator)
The outer radius is 10 cm and the inner radius is 9.5 cm.
The density of the metal is 11.3 g/cm^3.
Work out the mass of the pipe. Give your answer in kilograms correct to 3 significant figures.

Volume of metal = $\pi \times 10^2 \times 30 - \pi \times 9.5^2 \times 30 = \pi(100 - 90.25) \times 30$
$= 918.92$ cm^3
Mass = density × volume = $11.3 \times 918.92 = 10384$ g
= 10.4 kg (3 s.f.)
Answer: 10.4 kg

7 Inverse functions

$f(x) = \frac{7}{x - 4} - 5$

(a) Find $f^{-1}(x)$.
(b) Find the value of $f^{-1}(2)$.

(a) Let $y = \frac{7}{x - 4} - 5$, so $y + 5 = \frac{7}{x - 4}$

Make x the subject: $x - 4 = \frac{7}{y + 5}$

$x = \frac{7}{y + 5} + 4$

$f^{-1}(x) = \frac{7}{x + 5} + 4$

(b) $f^{-1}(2) = \frac{7}{2 - (-5)} + 4 = 5$

Answer: (a) $f^{-1}(x) = \frac{7}{x + 5} + 4$ (b) 5

8 Bounds: finding an unknown length

A rectangle has area 50 cm^2, correct to 2 significant figures.
Its length is 7.400000000000000355271367880050092935562133789062 5 cm, correct to 2 significant figures.
Work out the upper bound of the width of the rectangle. Give your answer correct to 3 significant figures. (Calculator)

Width = area ÷ length
Area: $49.5 \leq A < 50.5$. Length: $7.350000000000000355271367880050092935562133789062 5 \leq l < 7.450000000000000355271367880050092935562133789062 5$
UB width = UB area ÷ LB length = $50.5 \div 7.350000000000000355271367880050092935562133789062 5 = 6.87$
Answer: 6.87 cm

9 Rectangle under a parabola

The diagram shows the curve $y = 54 - 2x^2$. Rectangle ABCD has A, B on the x-axis and C, D on the curve. B is (x, 0).
(a) Show that the area of ABCD is $-4x^3 + 108x$.
(b) Find the maximum area of ABCD, showing that it is a maximum.

AB = 2x and BC = $54 - 2x^2$, so area = $2x(54 - 2x^2) = -4x^3 + 108x$

$\frac{dA}{dx} = -12x^2 + 108 = 0$, so $x^2 = 9$ and $x = 3$ ($x > 0$)

$\frac{d^2A}{dx^2} = -24x < 0$ when $x = 3$, so a maximum

Area = $108 \times 3 - 4 \times 3^3 = 216$

Answer: 216

10 Bounds of a compound expression

p = 13.2 and r = 5.6, each correct to 1 decimal place.
q = 3.67, correct to 2 decimal places.

Work out the lower bound of $\frac{p - q}{r}$. Give your answer correct to 3 significant figures. (Calculator)

For the lower bound use LB of the top and UB of the bottom
Top: LB p − UB q = 13.15 − 3.675 = 9.475
Bottom: UB r = 5.65
9.475 ÷ 5.65 = 1.68
Answer: 1.68