Day 176 **IGCSE 10-a-day: Model Solutions**

1 Tests of divisibility

The 4-digit number 181k is a multiple of 6.
k is a single digit.
Find all the possible values of k.

A multiple of 6 is even and a multiple of 3
The last digit k must be even
Digit sum = 1 + 8 + 1 + k = 10 + k
This must be a multiple of 3, so k = 2 or 5 or 8
Even values only
Answer: k = 2 or 8

2 Three-part ratios

Sofia, Oscar and Jamal share some money.
Sofia : Oscar = 6 : 2
Oscar : Jamal = 8 : 7
(a) Work out the ratio Sofia : Oscar : Jamal.
(b) They share £234 altogether. How much money does Oscar get?

(a) Make the Oscar part the same: 6 : 2 = 24 : 8 and 8 : 7 = 8 : 7
Sofia : Oscar : Jamal = 24 : 8 : 7
(b) Total parts = 39, so 1 part = 234 ÷ 39 = 6
Oscar gets 8 × 6 = £48
Answer: (a) 24 : 8 : 7 (b) £48

3 Show that dy/dx = ...

$y = \frac{(x+3)^2}{x}$ $(x \neq 0)$

(a) Show that $\frac{dy}{dx} = \frac{(x-3)(x+3)}{x^2}$.

(b) Find the gradient when x = 2.

Expand: $(x+3)^2 = x^2 + 6x + 9$, so $y = x + 6 + 9x^{-1}$

$\frac{dy}{dx} = 1 - \frac{9}{x^2} = \frac{x^2 - 9}{x^2} = \frac{(x-3)(x+3)}{x^2}$

When x = 2: $\frac{dy}{dx} = -\frac{5}{4}$

Answer: (b) $-\frac{5}{4}$

4 Composite with an inverse function

$f(x) = 2x - 1$ and $g(x) = 3x^2 - x - 5$.
(a) Find $f^{-1}g(x)$.
(b) Work out $f^{-1}g(2)$.

(a) $f^{-1}(x) = \frac{x+1}{2}$

$f^{-1}g(x) = \frac{3x^2 - x - 5 + 1}{2} = \frac{3x^2 - x - 4}{2}$

(b) g(2) = 5, so $f^{-1}g(2) = \frac{5+1}{2}$

Answer: (a) $\frac{3x^2 - x - 4}{2}$ (b) 3

5 Stationary points and their nature

$y = -x^3 - 6x^2 + 8$

(a) Find the coordinates of the stationary points.

(b) Use $\frac{d^2y}{dx^2}$ to decide whether each is a maximum or a minimum.

$\frac{dy}{dx} = -3x^2 - 12x = -3x(x + 4)$

$\frac{dy}{dx} = 0$ gives $x = -4$ or $x = 0$, so the points are $(-4, -24)$ and $(0, 8)$

$\frac{d^2y}{dx^2} = -6x - 12$

$x = -4$: $\frac{d^2y}{dx^2} = 12$, which is positive, so a minimum

$x = 0$: $\frac{d^2y}{dx^2} = -12$, which is negative, so a maximum

Answer: (a) $(-4, -24)$ and $(0, 8)$ (b) $(-4, -24)$ is a minimum, $(0, 8)$ is a maximum

6 Direct and inverse proportion

P is inversely proportional to the square of t.

When t = 3, P = 1.

(a) Find a formula for P in terms of t.

(b) Find the value of t when P = 9.

(a) $P = \frac{k}{t^2}$

Substitute t = 3 and P = 1: $1 = \frac{k}{3^2}$

$k = 9$

$P = \frac{9}{t^2}$

(b) $9 = \frac{9}{t^2}$, so $t^2 = \frac{9}{9} = 1$, so $t = 1$

Answer: (a) $P = \frac{9}{t^2}$ (b) $t = 1$

7 Standard form

Work out $(4.2 \times 10^{-4}) \div (3 \times 10^{-7})$

Give your answer in standard form.

$4.2 \div 3 = 1.4$

$10^{-4} \div 10^{-7} = 10^3$

$= 1.4 \times 10^3$

Answer: 1.4×10^3

8 Compound interest

£1,000 is invested for 3 years at 3% per year compound interest. (Calculator)

Work out the total amount at the end of 3 years.

Multiplier $= 1 + 3 \div 100 = 1.03$

Amount $= 1000 \times 1.03^3$

= £1,092.73

Answer: £1,092.73

9 Tangent to a circle

A circle has equation $x^2 + y^2 = 625$.
The point P$(-24, -7)$ lies on the circle.
(a) Find the equation of the tangent to the circle at P. Give your answer in the form ax + by = c, where a, b and c are integers.
(b) The tangent meets the x-axis at the point T. Write down the coordinates of T.

(a) The gradient of the radius OP is $\frac{7}{24}$

The tangent is perpendicular to the radius, so its gradient is $-1 \div \frac{7}{24} = -\frac{24}{7}$

$y - (-7) = (-\frac{24}{7})(x - (-24))$

Multiply out and rearrange: $24x + 7y = -625$
(b) On the x-axis y = 0, so $24x = -625$
$x = -\frac{625}{24}$, so T is $(-\frac{625}{24}, 0)$

Answer: (a) $24x + 7y = -625$ (b) T = $(-\frac{625}{24}, 0)$

10 Number of solutions of f(x) = k

The diagram shows a sketch of the curve $y = -x^3 - 6x^2 - 9x - 6$ and the line y = k.
The line meets the curve at three different points.
Find the range of possible values of k.

$\frac{dy}{dx} = -3x^2 - 12x - 9 = 0$ gives $x = -3$ and $x = -1$
$y = -6$ at $x = -3$ and $y = -2$ at $x = -1$
The line crosses three times when it lies between the two turning-point values
$-6 < k < -2$
Answer: $-6 < k < -2$