Day 175 **IGCSE 10-a-day: Model Solutions**

1 Quadrilateral: cosine rule, then area

In quadrilateral ABCD, AB = 6 cm, BC = 12 cm, CD = 8 cm,
angle ABC = 85° and angle ACD = 61°.
Work out the area of ABCD. Give your answer correct to 3 significant figures. (Calculator)

$AC^2 = 6^2 + 12^2 - 2 \times 6 \times 12 \times \cos 85°$ so AC = 12.940 cm
Area ABC = ½ × 6 × 12 × sin 85° = 35.86
Area ACD = ½ × 12.940 × 8 × sin 61° = 45.27
Total area = 35.86 + 45.27 = 81.1

Answer: 81.1 cm^2

2 Substitution

Work out the value of $\frac{2x - y}{z}$
when x = −1, y = −1 and z = −2.

Substitute the values: $\frac{2 \times (-1) - (-1)}{-2}$

$= \frac{-1}{-2}$

$= \frac{1}{2}$

Answer: $\frac{1}{2}$

3 Angle in a semicircle

AB is a diameter of a circle, centre O. C is a point on the circle. Angle BAC = 67°.
(a) Work out the size of angle ACB.
(b) Work out the size of angle COB.
The diagram is not drawn accurately.

(a) The angle in a semicircle is 90°, so angle ACB = 90°
(b) OA = OC (radii), so triangle OAC is isosceles and angle OCA = 67°
Angle COB is an exterior angle of triangle OAC: 67 + 67 = 134°
(or: the angle at the centre is twice the angle at the circumference)
Answer: (a) 90° (b) 134°

4 Two numbers: maximise or minimise

The positive numbers x and y satisfy x + y = 6.
Find the greatest possible value of x^2y, showing that it is the greatest.

$y = 6 - x$, so $x^2y = -x^3 + 6x^2$
$\frac{d}{dx}(x^2y) = -3x^2 + 12x = 0$, so $x(12 - 3x) = 0$ and x = 4 (x > 0)
Second derivative = −6x + 12 = −12 < 0, so a maximum
y = 2, so the greatest value is $4^2 \times 2 = 32$
Answer: 32

5 nth term of a quadratic sequence

Here are the first five terms of a quadratic sequence.
16, 35, 64, 103, 152
Find an expression for the nth term.

First differences: 19, 29, 39, 49. Second differences: 10, 10, 10
Second difference = 2a, so a = 5
Subtract $5n^2$: 11, 15, 19, 23, 27
This is linear: 4n + 7
nth term = $5n^2 + 4n + 7$
Answer: $5n^2 + 4n + 7$

6 Increasing and decreasing with powers

$f(x) = x^{\frac{3}{2}} - 12x$ for $x > 0$

(a) Find the range of values of x for which f is a decreasing function.

(b) Find the least value of f(x).

$f'(x) = \frac{3}{2}x^{\frac{1}{2}} - 12$

$f'(x) < 0$ when $\frac{3}{2}x^{\frac{1}{2}} < 12$, so $x^{\frac{1}{2}} < 8$

Square both sides: $x < 64$, so $0 < x < 64$

f decreases then increases, so the least value is at $x = 64$: $f = 64^{\frac{3}{2}} - 12 \times 64 = 512 - 768 = -256$

Answer: (a) $0 < x < 64$ (b) -256

7 Fractional indices

Given that $x > 0$, solve $x^{\frac{3}{2}} = 27$

Raise both sides to the power $\frac{2}{3}$

$x = (27)^{\frac{2}{3}}$

$x = 9$

Answer: $x = 9$

8 Three stationary points of a quartic

$y = 2x^4 - 36x^2 + 8$

Find the coordinates of the stationary points of the curve and determine the nature of each.

$\frac{dy}{dx} = 8x^3 - 72x = 8x(x - 3)(x + 3)$

$\frac{dy}{dx} = 0$ gives $x = -3$, $x = 0$ or $x = 3$

Points: $(-3, -154)$, $(0, 8)$, $(3, -154)$

$\frac{d^2y}{dx^2} = 24x^2 - 72$

$x = -3$: $\frac{d^2y}{dx^2} = 144$; $x = 0$: $\frac{d^2y}{dx^2} = -72$; $x = 3$: $\frac{d^2y}{dx^2} = 144$

So $(-3, -154)$ is a minimum, $(0, 8)$ is a maximum, $(3, -154)$ is a minimum

Answer: $(-3, -154)$ minimum; $(0, 8)$ maximum; $(3, -154)$ minimum

9 Reverse percentages

The price of a tablet is increased by 5%.

Later the new price is reduced by 20% in a sale.

The sale price is £218.40.

Work out the price before the increase. (Calculator)

Multipliers: 1.05 and 0.8

Overall multiplier = $1.05 \times 0.8 = 0.84$

Original $\times$ 0.84 = £218.40

Original = £218.40 ÷ 0.84 = £260

Answer: £260

10 Equation of a circle

A circle has centre C(5, −5) and passes through the point P(6, 1).

Find an equation of the circle.

$r^2 = CP^2 = (6 - 5)^2 + (1 - (-5))^2$

$= 1 + 36 = 37$

$(x - 5)^2 + (y + 5)^2 = 37$

Answer: $(x - 5)^2 + (y + 5)^2 = 37$