Day 174 **IGCSE 10-a-day: Model Solutions**

1 Mean of a changing set

The mean of 9 numbers is 81.
A new number is added and the mean of all 10 numbers is 75.9.
Work out the new number.

Total of the 9 numbers = $9 \times 81 = 729$
New total = $10 \times 75.9 = 759$
New number = $759 - 729 = 30$
Answer: 30

2 Velocity and acceleration

A particle P moves in a straight line. After t seconds its displacement from a fixed point O is s metres, where $s = -t^3 - 8t^2 + 8t$ $(t > 0)$
Find (a) the velocity of P, (b) the acceleration of P, when t = 3.

$v = \frac{ds}{dt} = -3t^2 - 16t + 8$

$a = \frac{dv}{dt} = \frac{d^2s}{dt^2} = -6t - 16$

When t = 3: $v = -67$ and $a = -34$
Answer: (a) -67 m/s (b) -34 m/s^2

3 Parallelogram and midpoint with vectors

ABCD is a parallelogram.
A = $(-1, 1)$, B = $(-2, 4)$ and D = $(-4, -1)$.
Find the coordinates of C and of the point where the diagonals meet.

$\mathbf{BC} = \mathbf{AD} = D - A = \begin{pmatrix} -3 \\ -2 \end{pmatrix}$

C = B + **AD** = $(-5, 2)$
The diagonals bisect each other, so they meet at the midpoint of AC
$(\frac{-1 + (-5)}{2}, \frac{1 + 2}{2}) = (-3, \frac{3}{2})$

Answer: C = $(-5, 2)$, diagonals meet at $(-3, \frac{3}{2})$

4 Expand then differentiate

$y = x(x - 1)(x - 3)$
Find $\frac{dy}{dx}$.

Expand: $y = x^3 - 4x^2 + 3x$
$x^3 \to 3x^2$, $-4x^2 \to -8x$, $3x \to 3$
$\frac{dy}{dx} = 3x^2 - 8x + 3$

Answer: $\frac{dy}{dx} = 3x^2 - 8x + 3$

5 Area and quadratic equations

The diagram shows a rectangle of length $(3x - 2)$ cm, width $(x + 3)$ cm and area 300 cm^2.
(a) Show that $3x^2 + 7x - 306 = 0$.
(b) Work out the perimeter of the rectangle.

(a) $(3x - 2)(x + 3) = 300$
Expand: $3x^2 + 7x - 6 = 300$, so $3x^2 + 7x - 306 = 0$
(b) Factorise: $(x - 9)(3x + 34) = 0$
$x = 9$ ($x = -\frac{34}{3}$ is not possible as a length would be negative)
Length = 25 cm and width = 12 cm
Perimeter = $2 \times (25 + 12) = 74$ cm
Answer: (b) 74 cm

6 Index equations

Solve $4^{4x+2} = 2^4$

Write both sides as powers of 2: $4 = 2^2$
Equate the powers: $8x + 4 = 4$
$8x = 0$, so $x = 0$
Answer: $x = 0$

7 Volume and surface area of composite solids

A solid cylinder of radius 6 cm and height 11 cm has a hemispherical hollow of the same radius cut out of the top.
Work out the volume of the solid that is left. Give your answer in terms of π.

Cylinder: $\pi \times 6^2 \times 11 = 396\pi$

Hemisphere: $\frac{2}{3} \times \pi \times 6^3 = 144\pi$

Volume left = $396\pi - 144\pi = 252\pi$ cm^3
Answer: 252π cm^3

8 Describe a graph transformation

Describe fully the single transformation that maps the graph of y = f(x) onto the graph of
(a) y = f(−x)
(b) y = f(x) − 1

(a) Changing the sign of every x value flips the graph over the y-axis
(b) Subtracting 1 from f(x) moves the graph down 1 units

Answer: (a) A reflection in the y-axis (b) A translation by the vector $\begin{pmatrix} 0 \\ -1 \end{pmatrix}$

9 Functions that are their own inverse

f(x) = $\frac{2x-3}{x-2}$ where $x \neq 2$
(a) Show that ff(x) = x.
(b) Write down $f^{-1}(x)$.

(a) Replace x in f(x) by f(x), then multiply the top and the bottom by $(x - 2)$
$ff(x) = \frac{2(2x-3)+(-3)(x-2)}{(2x-3)-2(x-2)}$
Top = $4x - 6 - 3x + 6 = x$
Bottom = $2x - 3 - 2x + 4 = 1$
$ff(x) = \frac{x}{1} = x$

(b) ff(x) = x, so f is its own inverse: $f^{-1}(x) = \frac{2x-3}{x-2}$

Answer: (b) $f^{-1}(x) = \frac{2x-3}{x-2}$

10 Curves that touch

The curves $y = 2x^2 + 2x + 4$ and $y = 3x^2 + 8x + 13$ touch at one point.
Find the coordinates of this point and the equation of the common tangent.

Equate the two expressions: $x^2 + 6x + 9 = 0$
That is $(x + 3)^2 = 0$ with a repeated root $x = -3$
y = 16, so the point is (−3, 16)
$\frac{dy}{dx} = 4x + 2$ on the first curve: gradient −10
Tangent: $y = -10x - 14$
Answer: (−3, 16), $y = -10x - 14$