Day 173 **IGCSE 10-a-day: Model Solutions**

1 Mean of a changing set

The mean of 9 numbers is 133.
One number is removed and the mean of the remaining 8 numbers is 145.25.
Work out the number that was removed.

Total of the 9 numbers = $9 \times 133 = 1197$
Total of the 8 remaining = $8 \times 145.25 = 1162$
Number removed = $1197 - 1162 = 35$
Answer: 35

2 Find unknown coefficients from gradients

The curve $y = x^3 + ax^2 + bx$ has gradient 5 when $x = 1$ and gradient 25 when $x = 3$.
Find the values of a and b.

$\frac{dy}{dx} = 3x^2 + 2ax + b$
$x = 1$: $2a + b = 2$
$x = 3$: $6a + b = -2$
Subtract and solve: $a = -1$
$b = 4$
Answer: $a = -1$, $b = 4$

3 Comparing costs

Hire Shop A has a delivery charge of £4 plus £5.50 per day.
Hire Shop B has a delivery charge of £10 plus £5 per day.
(a) Work out the number of days for which the two costs are the same.
(b) Which is cheaper for 20 days, and by how much?

(a) Let n be the number of days. Working in pounds, the costs are equal when $4 + 5.50n = 10 + 5n$
$0.50n = 6$, so $n = 12$
(b) Hire Shop A: £4 + £5.50 × 20 = £114
Hire Shop B: £10 + £5 × 20 = £110
Hire Shop B is cheaper by £4
Answer: (a) 12 (b) Hire Shop B, by £4

4 Transformation by a matrix: image and description

$M = \begin{pmatrix} 4 & 0 \\ 0 & 4 \end{pmatrix}$.
(a) Find the coordinates of the image of the point $(-1, 1)$ under the transformation represented by M.
(b) Describe fully the single transformation represented by M.

(a) $\begin{pmatrix} 4 & 0 \\ 0 & 4 \end{pmatrix}\begin{pmatrix} -1 \\ 1 \end{pmatrix} = \begin{pmatrix} -4 \\ 4 \end{pmatrix}$
The image is $(-4, 4)$
(b) The first column is where (1, 0) goes and the second column is where (0, 1) goes
M represents an enlargement, scale factor 4, centre the origin
Answer: (a) $(-4, 4)$ (b) Enlargement, scale factor 4, centre the origin

5 Areas of similar triangles

In the diagram, DE is parallel to BC. AD = 3 cm, DB = 3 cm and the area of triangle ADE is 18 cm^2.
Work out the area of the trapezium DBCE.
The diagram is not drawn accurately.

Triangles ADE and ABC are similar. AB = 3 + 3 = 6 cm
Length scale factor = $6 \div 3 = 2$
Area scale factor = $(2)^2 = 4$
Area of triangle ABC = $18 \times 4 = 72$ cm^2
Trapezium = $72 - 18 = 54$ cm^2
Answer: 54 cm^2

6 Linear programming: vertices

x and y satisfy $x \geq 0$, $y \geq 0$,
$2x + 4y \leq 52$ and $5x + 5y \leq 90$.
Find the greatest value of $P = 2x + 3y$ and the values of x and y that give it.

Draw the lines. The feasible region has vertices (0, 0), (0, 13), (10, 8), (18, 0)
Values of P: (0, 0): 0, (0, 13): 39, (10, 8): 44, (18, 0): 36
Greatest value of P is 44 at (10, 8)
Answer: P = 44 when x = 10 and y = 8

7 Fractions removed from a ratio

The ratio of blue to green sweets in a tin is 5 : 6.
Then $\frac{1}{5}$ of the blue and $\frac{1}{4}$ of the green are taken out.
Work out the ratio of blue to green now.

Left: $\frac{4}{5} \times 5 = 4$ of the blue part, $\frac{3}{4} \times 6 = \frac{9}{2}$ of the green part
$4 : \frac{9}{2}$
Multiply by 2: 8 : 9
Answer: 8 : 9

8 Area of triangle from a tangent

The tangent to the curve $y = 2x^2 - x + 6$ at the point where x = 0 meets the axes at A and B.
Work out the area of triangle OAB, where O is the origin.

$y = 6$ and $\frac{dy}{dx} = 4x - 1$, so the gradient is -1
Tangent: $y = -x + 6$
y-axis: (0, 6). x-axis: $0 = -x + 6$ so $x = 6$
Area $= \frac{1}{2} \times 6 \times 6$
$= 18$
Answer: 18 square units

9 Error intervals

The height of a tower is 360 m, correct to the nearest 5.
Write down the error interval for the height of a tower, using h.

The value has been rounded to the nearest 5
Half of this is 2.5, so go 2.5 below and 2.5 above
$360 - 2.5 = 357.5$ and $360 + 2.5 = 362.5$
$357.5 \leq h < 362.5$
Answer: $357.5 \leq h < 362.5$

10 Tangent meets the curve again (cubic)

The tangent to the curve $y = x^3 - x^2 - 2x - 4$ at the point P where x = 0 meets the curve again at Q.
Find the coordinates of Q.

$\frac{dy}{dx} = 3x^2 - 2x - 2$, so at P the gradient is -2 and $y = -4$
Tangent: $y = -2x - 4$
Intersection: $x^3 - x^2 = 0$
x = 0 is a repeated root, so the roots are 0, 0 and x_Q
Sum of roots = 1, so $0 + x_Q = 1$ and $x_Q = 1$
$y = -6$
Answer: (1, −6)