Day 172 **IGCSE 10-a-day: Model Solutions**

1 Compound interest

Sofia invests £3,500 at 5% per year compound interest.
After how many whole years will the investment first be worth more than £5,100?
(Calculator)

Multiplier = 1.05

After 7 years: $3500 \times 1.05^7 = 4924.85$

After 8 years: $3500 \times 1.05^8 = 5171.09$

5171.09 is the first value above £5,100

Answer: 8 years

2 Turning point of a quadratic by differentiation

$f(x) = x^2 - 6x - 6$

Use differentiation to find the coordinates of the turning point of the graph of y = f(x). State whether it is a maximum or a minimum.

$f'(x) = 2x - 6$

$f'(x) = 0$: $2x - 6 = 0$, so $x = 3$

$f(3) = -15$

$f''(x) = 2$, which is positive, so it is a minimum

Answer: (3, −15), minimum

3 Laws of indices with algebra

Simplify $(9x^6y^6)^{\frac{3}{2}}$

Raise each part to the power $\frac{3}{2}$

$9^{\frac{3}{2}} = (\sqrt{9})^3 = 3^3 = 27$

$(x^6)^{\frac{3}{2}} = x^9$ and $(y^6)^{\frac{3}{2}} = y^9$

$= 27x^9y^9$

Answer: $27x^9y^9$

4 3D trigonometry: cuboid

ABCDEFGH is a cuboid. AB = 10 cm, BC = 14 cm and CG = 5 cm.
(a) Work out the length of AG.
(b) Work out the size of the angle between AG and the face ABFE.
Give answers to 3 significant figures or 1 decimal place. (Calculator)

(a) $AG^2 = 10^2 + 14^2 + 5^2 = 321$, so AG = 17.92 cm

(b) G projects onto F on face ABFE (FG is perpendicular to the face), so the angle is GAF

$AF^2 = 10^2 + 5^2 = 125$ and FG = 14

$\tan GAF = \frac{14}{\sqrt{125}}$, angle = 51.4°

Answer: (a) 17.9 cm (b) 51.4°

5 Stationary points and their nature

$y = x^3 - 12x - 3$

(a) Find the coordinates of the stationary points.

(b) Use $\frac{d^2y}{dx^2}$ to decide whether each is a maximum or a minimum.

$\frac{dy}{dx} = 3x^2 - 12 = 3(x + 2)(x - 2)$

$\frac{dy}{dx} = 0$ gives $x = -2$ or $x = 2$, so the points are $(-2, 13)$ and $(2, -19)$

$\frac{d^2y}{dx^2} = 6x$

$x = -2$: $\frac{d^2y}{dx^2} = -12$, which is negative, so a maximum

$x = 2$: $\frac{d^2y}{dx^2} = 12$, which is positive, so a minimum

Answer: (a) $(-2, 13)$ and $(2, -19)$ (b) $(-2, 13)$ is a maximum, $(2, -19)$ is a minimum

6 Solving quadratics by factorising

Solve $2x^2 + 7x = -6$

Rearrange so that one side is 0: $2x^2 + 7x + 6 = 0$

Factorise: $(2x + 3)(x + 2) = 0$

$2x + 3 = 0$ or $x + 2 = 0$

$x = -\frac{3}{2}$ or $x = -2$

Answer: $x = -2$ or $x = -\frac{3}{2}$

7 Tangent meets the axes

The diagram shows the curve $y = 2x^2 - 5x - 4$ and the tangent at P$(2, -6)$, meeting the axes at A and B.

(a) Find an equation of the tangent at P.

(b) Work out the area of triangle OAB.

$\frac{dy}{dx} = 4x - 5$, so at $x = 2$ the gradient is 3

$y - (-6) = 3(x - 2)$, so $y = 3x - 12$

B: $x = 0$ gives $y = -12$. A: $y = 0$ gives $x = 4$

Area = ½ × 4 × 12 = 24

Answer: (a) $y = 3x - 12$ (b) 24

8 Tangent to a circle

A circle has equation $x^2 + y^2 = 625$.

The point P$(-7, -24)$ lies on the circle.

(a) Find the equation of the tangent to the circle at P. Give your answer in the form $ax + by = c$, where a, b and c are integers.

(b) The tangent meets the x-axis at the point T. Write down the coordinates of T.

(a) The gradient of the radius OP is $\frac{24}{7}$

The tangent is perpendicular to the radius, so its gradient is $-1 \div \frac{24}{7} = -\frac{7}{24}$

$y - (-24) = (-\frac{7}{24})(x - (-7))$

Multiply out and rearrange: $7x + 24y = -625$

(b) On the x-axis $y = 0$, so $7x = -625$

$x = -\frac{625}{7}$, so T is $(-\frac{625}{7}, 0)$

Answer: (a) $7x + 24y = -625$ (b) T $= (-\frac{625}{7}, 0)$

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9 Bounds of a compound expression

p = 10.3 and r = 7.4, each correct to 1 decimal place.
q = 2.95, correct to 2 decimal places.

Work out the upper bound of $\frac{p + q}{r}$. Give your answer correct to 3 significant figures. (Calculator)

For the upper bound use UB of the top and LB of the bottom
Top: UB p + UB q = 10.35 + 2.955 = 13.305
Bottom: LB r = 7.35
13.305 ÷ 7.35 = 1.81
Answer: 1.81

10 Bounds and accuracy

a = 64 and b = 3.29, both correct to 3 significant figures.
Work out a ÷ b.
Write down your answer to a suitable degree of accuracy.
You must show your working and give a reason for your answer. (Calculator)

$63.95 \leq a < 64.05$; $3.285 \leq b < 3.295$
Lower bound = 63.95 ÷ 3.295 = 19.408...
Upper bound = 64.05 ÷ 3.285 = 19.498...
Both bounds round to 19 to 2 significant figures, but not to 3 significant figures
Answer: 19 (2 s.f.)