Day 171 **IGCSE 10-a-day: Model Solutions**

1 Substitution

Work out the value of $(x - y)^2 - (x + y)^2$
when $x = -2$, $y = -2$ and $z = -3$.

Substitute the values: $(-2 - (-2))^2 - (-2 + (-2))^2$
$= 0 - 16$
$= -16$
Answer: -16

2 Stationary point with a 1/x term

A curve has equation $y = x^2 - \frac{16}{x}$.

(a) Show that the curve has a stationary point at $x = -2$.
(b) Find the y coordinate of this point and decide whether it is a maximum or a minimum.

$\frac{dy}{dx} = 2x + \frac{16}{x^2}$

When $x = -2$: $\frac{dy}{dx} = -4 - (-4) = 0$

$y = 12$

$\frac{d^2y}{dx^2} = 2 - \frac{32}{x^3}$

At $x = -2$: $\frac{d^2y}{dx^2} = 6$, which is positive, so a minimum

Answer: y = 12, minimum

3 Venn diagram: reading set notation

The Venn diagram shows information about 39 pupils.
A: walk to school B: cycle to school
(a) Find $n(A' \cap B)$.
(b) Find $n(A')$.

(a) $n(A' \cap B) = 11$
(b) $n(A') = 11 + 4 = 15$
Answer: (a) 11 (b) 15

4 Equation of a normal

A curve has equation $y = x^2 - 6x - 6$.
Find the equation of the normal to the curve at the point where $x = -1$.

When $x = -1$, $y = 1$ and $\frac{dy}{dx} = -8$

Gradient of the normal $= -1 \div (-8) = \frac{1}{8}$

$y - 1 = \frac{1}{8}(x + 1)$

$y = \frac{1}{8}x + \frac{9}{8}$

Answer: $y = \frac{1}{8}x + \frac{9}{8}$

5 HCF and LCM

(a) Write 99 and 432 as products of prime factors.
(b) Find the HCF and the LCM of 99 and 432.
Give your answers as numbers.

(a) $99 = 3^2 \times 11$ and $432 = 2^4 \times 3^3$
(b) HCF $= 3^2 = 9$
LCM $= 2^4 \times 3^3 \times 11 = 4752$
Answer: (a) $3^2 \times 11$; $2^4 \times 3^3$ (b) HCF = 9, LCM = 4752

6 Cosine rule and area of a triangle

The diagram shows triangle ABC. (Calculator)
(a) Work out angle ACB, marked θ, to 1 decimal place.
(b) Work out the area of triangle ABC to 3 significant figures.
The diagram is not drawn accurately.

(a) $\cos\theta = \frac{5^2 + 5^2 - 6^2}{2 \times 5 \times 5} = \frac{14}{50}$

$\theta = 73.7°$ (1 d.p.)

(b) Area $= \frac{1}{2} \times 5 \times 5 \times \sin 73.7°$

$= 12.0\ \text{cm}^2$ (3 s.f.)

Answer: (a) 73.7° (b) 12.0 cm^2

7 Number of solutions of f(x) = k

The diagram shows a sketch of the curve $y = 2x^3 - 6x^2$ and the line $y = k$.
The line meets the curve at three different points.
Find the range of possible values of k.

$\frac{dy}{dx} = 6x^2 - 12x = 0$ gives $x = 0$ and $x = 2$

$y = 0$ at $x = 0$ and $y = -8$ at $x = 2$

The line crosses three times when it lies between the two turning-point values

$-8 < k < 0$

Answer: $-8 < k < 0$

8 Ratio and expressions

$x : y = 8 : 3$
Work out the ratio $(3x - 2y) : (3x + 5y)$
Give your answer in its simplest form.

$x : y = 8 : 3$, so let $x = 8t$ and $y = 3t$

$3(8t) - 2(3t) = 18t$

$3(8t) + 5(3t) = 39t$

$18t : 39t = 6 : 13$

Answer: 6 : 13

9 Finding a matrix from MA = B

$A = \begin{pmatrix} 2 & 3 \\ -2 & -2 \end{pmatrix}$ and $B = \begin{pmatrix} -8 & -11 \\ 4 & 7 \end{pmatrix}$.
MA = B, where M is a 2 × 2 matrix.
Find M.

MA = B so $M = BA^{-1}$

$A^{-1} = \frac{1}{2}\begin{pmatrix} -2 & -3 \\ 2 & 2 \end{pmatrix}$

$M = \begin{pmatrix} -8 & -11 \\ 4 & 7 \end{pmatrix}\frac{1}{2}\begin{pmatrix} -2 & -3 \\ 2 & 2 \end{pmatrix} = \begin{pmatrix} -3 & 1 \\ 3 & 1 \end{pmatrix}$

Answer: $\mathbf{M} = \begin{pmatrix} \mathbf{-3} & \mathbf{1} \\ \mathbf{3} & \mathbf{1} \end{pmatrix}$

10 Speed and a moving train

A train travels at a constant speed of 72 km/h.
It takes 35 seconds to go completely through a tunnel which is 630 m long.
Work out the length of the train, in metres.

Speed = 72 km/h = 72,000 ÷ 3600 = 20 m/s
Distance travelled in 35 s = 20 × 35 = 700 m
This is the length of the tunnel plus the length of the train
Train = 700 − 630 = 70 m
Answer: 70 m