Day 170 **IGCSE 10-a-day: Model Solutions**

1 Inverse of a rational function

$f(x) = \frac{3x - 6}{2x + 4}$, $x \neq -2$.

Find $f^{-1}(x)$.

$y = \frac{3x - 6}{2x + 4}$, so $y(2x + 4) = 3x - 6$

$2xy + 4y = 3x - 6$

$2xy - 3x = -6 - 4y$

$x(2y - 3) = -6 - 4y$, so $x = \frac{-6 - 4y}{2y - 3}$

$f^{-1}(x) = \frac{-6 - 4x}{2x - 3}$

Answer: $f^{-1}(x) = \frac{-6 - 4x}{2x - 3}$

2 Tangent at a turning point

The graph of $y = -x^2 - 2x - 3$ has a turning point at P.
Find the equation of the tangent to the curve at P.

$\frac{dy}{dx} = -2x - 2$

At P, $\frac{dy}{dx} = 0$, so $-2x - 2 = 0$ and $x = -1$

$y = -2$ at P

The gradient is 0, so the tangent is horizontal: $y = -2$

Answer: $y = -2$

3 Tests of divisibility

The 4-digit number 61k0 is a multiple of 9.
k is a single digit.
Find all the possible values of k.

A number is a multiple of 9 if its digit sum is a multiple of 9

Digit sum = 6 + 1 + 0 + k = 7 + k

This must be a multiple of 9, so k = 2

Answer: k = 2

4 Sine rule

The diagram shows triangle ABC. (Calculator)
Angle BAC = 66° and angle ABC is acute.
Work out the size of angle ABC, marked θ. Give your answer correct to 1 decimal place.
The diagram is not drawn accurately.

Use the sine rule: $\frac{\sin \theta}{AC} = \frac{\sin A}{BC}$

$\frac{\sin \theta}{14} = \frac{\sin 66°}{15}$

$\sin \theta = \frac{14 \times \sin 66°}{15} = 0.85264$

$\theta = \sin^{-1}(0.85264) = 58.5°$ (1 d.p.)

Answer: $\theta = 58.5°$

5 Show particle never at rest

A particle moves in a straight line. After t seconds its displacement from a fixed point is s metres, where

$s = t^3 - 12t^2 + 50t + 5$

(a) Show that the particle is never at rest.

(b) Find the least speed of the particle and the time it occurs.

$v = \frac{ds}{dt} = 3t^2 - 24t + 50 = 3(t - 4)^2 + 2$

A square is never negative, so $v \geq 2 > 0$ and P is never at rest

Least value of v occurs when t = 4, giving 2

Answer: (b) 2 m/s when t = 4 s

6 Collinear points with vectors

$\mathbf{OA} = -2\mathbf{b}$, $\mathbf{OB} = 3\mathbf{a} + \mathbf{b}$ and $\mathbf{OC} = 4\mathbf{a} + 2\mathbf{b}$.

Show that A, B and C lie on a straight line.

$\mathbf{AB} = \mathbf{OB} - \mathbf{OA} = 3\mathbf{a} + 3\mathbf{b}$

$\mathbf{AC} = \mathbf{OC} - \mathbf{OA} = 4\mathbf{a} + 4\mathbf{b}$

$\mathbf{AC} = \frac{4}{3}\mathbf{AB}$

AC is a multiple of AB and they share the point A, so A, B and C are collinear

Answer: $\mathbf{AC} = \frac{4}{3}\mathbf{AB}$, so A, B, C are collinear

7 Fractions removed from a ratio

The ratio of men to women people at a meeting is 7 : 9.

Then $\frac{1}{3}$ of the men and $\frac{2}{3}$ of the women are taken out.

Work out the ratio of men to women now.

Left: $\frac{2}{3} \times 7 = \frac{14}{3}$ of the men part, $\frac{1}{3} \times 9 = 3$ of the women part

$\frac{14}{3} : 3$

Multiply by 3: 14 : 9

Answer: 14 : 9

8 Pythagoras and quadratic equations

The lengths of the sides of a right-angled triangle are (x + 8) cm, (x + 1) cm and (x + 9) cm.

The longest side is (x + 9) cm.

(a) Show that $x^2 - 16 = 0$

(b) Hence work out the area of the triangle.

(a) Pythagoras: $(x + 8)^2 + (x + 1)^2 = (x + 9)^2$

Expand: $x^2 + 16x + 64 + x^2 + 2x + 1 = x^2 + 18x + 81$

Rearrange: $x^2 - 16 = 0$

(b) Factorise: $(x - 4)(x + 4) = 0$, so x = 4 (the other root, x = −4, does not give valid side lengths)

The shorter sides are 12 cm and 5 cm, so area = $\frac{1}{2} \times 12 \times 5 = 30$ cm^2

Answer: (b) 30 cm^2

9 Perimeter and area of a running track

The diagram shows a running track made from a rectangle and two semicircles.
(a) Work out the total length of the boundary of the track.
(b) Work out the area enclosed by the track. Give your answers in terms of π.
The diagram is not drawn accurately.

(a) The two semicircles make one full circle of radius 30: circumference = $2 \times \pi \times 30 = 60\pi$
The two straight sides: $2 \times 60 = 120$
Boundary = $(120 + 60\pi)$ m

(b) The rectangle is 60 m by $2 \times 30 = 60$ m, so its area is $60 \times 60 = 3600$. Circle: $\pi \times 30^2 = 900\pi$
Area = $(3600 + 900\pi)$ m^2

Answer: (a) $(120 + 60\pi)$ m (b) $(3600 + 900\pi)$ m^2

10 Fastest rate of growth

The number of fish in a lake, N, after t months is modelled by
$N = -t^3 + 6t^2 + 1000$ for $0 \leq t \leq 8$
Find the time at which N is increasing most rapidly, and the rate of increase at this time.

$\frac{dN}{dt} = -3t^2 + 12t$

Rate is greatest when $\frac{d^2N}{dt^2} = -6t + 12 = 0$, so $t = 2$

$\frac{d^2N}{dt^2}$ is decreasing (gradient -6), so the rate is a maximum

$\frac{dN}{dt} = 12$ when $t = 2$

Answer: t = 2, rate 12 per month