Day 17 **IGCSE 10-a-day: Model Solutions**

1 Equation of a normal

A curve has equation $y = 2x^2 - 6x + 1$.
Find the equation of the normal to the curve at the point where $x = 0$.

When $x = 0$, $y = 1$ and $\frac{dy}{dx} = -6$

Gradient of the normal $= -1 \div (-6) = \frac{1}{6}$

$y - 1 = \frac{1}{6}x$

$y = \frac{1}{6}x + 1$

Answer: $y = \frac{1}{6}x + 1$

2 Chords and Pythagoras in circles

AB is a chord of a circle, centre O. M is the midpoint of AB. (Calculator)
(a) Work out the length of the chord AB.
(b) Work out angle AOB, to 1 decimal place.
The diagram is not drawn accurately.

(a) $AM^2 = OA^2 - OM^2 = 17^2 - 8^2 = 225$
AM = 15 cm
AB = 2 × 15 = 30 cm

(b) sin(angle AOM) $= \frac{15}{17}$

Angle AOM = 61.93°, so angle AOB = 2 × 61.93 = 123.9°
Answer: (a) 30 cm (b) 123.9°

3 Percentage change

The price of a phone is decreased by 25%.
The new price is then decreased by 15%.
Work out the overall percentage change in the price.

Multipliers: 0.75 and 0.85
Overall multiplier = 0.75 × 0.85 = 0.6375
0.6375 − 1 = −0.3625, so an overall decrease of 36.25%
Answer: 36.25% decrease

4 Rates of work

Tap A can fill an empty tank in 8 minutes.
Tap B can fill an empty tank in 24 minutes.
Both are turned on at the same time.
How long do they take to fill an empty tank?

In 1 minute: Tap A does $\frac{1}{8}$ and Tap B does $\frac{1}{24}$

Together: $\frac{1}{8} + \frac{1}{24} = \frac{32}{192}$

Time $= \frac{192}{32} = 6$ minutes

Answer: 6 minutes

5 Simplifying algebraic fractions

Simplify fully $\frac{x^2 - 81}{x^2 + 15x + 54}$

Factorise the top: $(x - 9)(x + 9)$
Factorise the bottom: $(x + 9)(x + 6)$
Cancel the common factor $(x + 9)$: $\frac{x - 9}{x + 6}$

Answer: $\mathbf{\frac{x - 9}{x + 6}}$

6 Describe a graph transformation

Describe fully the single transformation that maps the graph of $y = f(x)$ onto the graph of
(a) $y = 3f(x)$
(b) $y = f(x + 7)$

(a) Every y value is multiplied by 3
(b) Replacing x by $x + 7$ moves the graph 7 units to the left

Answer: (a) A stretch parallel to the y-axis with scale factor 3 (b) A translation by the vector $\begin{pmatrix} -7 \\ 0 \end{pmatrix}$

7 Intersection and gradient

The curve $y = -2x^2 + x + 5$ and the line $y = -5x + 5$ intersect at two points A and B, where A has the smaller x coordinate.
(a) Find the coordinates of A and B.
(b) Find the gradient of the curve at A and at B.

$-2x^2 + x + 5 = -5x + 5$ gives $-2x^2 + 6x = 0$, so $-2x(x - 3) = 0$
$x = 0$ or $x = 3$, giving A (0, 5) and B (3, −10)
$\frac{dy}{dx} = -4x + 1$: at A it is 1, at B it is −11

Answer: (a) A (0, 5), B (3, −10) (b) 1 at A, −11 at B

8 Algebraic fractions: multiplying and dividing

Simplify fully $\frac{x^2 + 5x + 6}{x + 1} \div \frac{x + 3}{x^2 + 7x + 6}$

Factorise: $\frac{(x + 2)(x + 3)}{(x + 1)} \div \frac{(x + 3)}{(x + 1)(x + 6)}$

Turn the second fraction upside down and multiply: $\frac{(x + 2)(x + 3)}{(x + 1)} \times \frac{(x + 1)(x + 6)}{(x + 3)}$

Cancel $(x + 3)$ and $(x + 1)$
$= (x + 2)(x + 6)$

Answer: $(x + 2)(x + 6)$

9 Maximum revenue

A company sells x thousand items each month. The price of each item is £$42 - 3x$.
The monthly revenue is £R thousand.
(a) Show that $R = 42x - 3x^2$.
(b) Find the maximum monthly revenue, showing that it is a maximum.

$R = x \times (42 - 3x) = 42x - 3x^2$
$\frac{dR}{dx} = -6x + 42 = 0$ gives $x = 7$
$\frac{d^2R}{dx^2} = -6 < 0$, so a maximum
$R = 42 \times 7 - 3 \times 7^2 = 147$

Answer: £147 thousand

10 Simultaneous equations with an inverse matrix

$M = \begin{pmatrix} -2 & -4 \\ 2 & 3 \end{pmatrix}$.

(a) Find M^{-1}.

(b) Use your answer to (a) to solve

$-2x - 4y = -18$

$2x + 3y = 12$

(a) det M = 2 so $M^{-1} = \frac{1}{2}\begin{pmatrix} 3 & 4 \\ -2 & -2 \end{pmatrix}$

(b) The equations are $\begin{pmatrix} -2 & -4 \\ 2 & 3 \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} -18 \\ 12 \end{pmatrix}$

$\begin{pmatrix} x \\ y \end{pmatrix} = \frac{1}{2}\begin{pmatrix} 3 & 4 \\ -2 & -2 \end{pmatrix}\begin{pmatrix} -18 \\ 12 \end{pmatrix} = \begin{pmatrix} -3 \\ 6 \end{pmatrix}$

Answer: (a) $\frac{1}{2}\begin{pmatrix} 3 & 4 \\ -2 & -2 \end{pmatrix}$ **(b) x = −3, y = 6**