Day 169 **IGCSE 10-a-day: Model Solutions**

1 Tangent to a curve with negative or fractional powers

A curve has equation $y = -3x^2 + \frac{3}{x}$.

Find the equation of the tangent to the curve at the point where x = 2.

$\frac{dy}{dx} = -6x - 3x^{-2}$

At x = 2: $y = -\frac{21}{2}$ and gradient $= -\frac{51}{4}$

$y + \frac{21}{2} = -\frac{51}{4}(x - 2)$

$y = -\frac{51}{4}x + 15$

Answer: $y = -\frac{51}{4}x + 15$

2 Time zones

A flight leaves London at 14:05 (London time).
It takes 12 hours 35 minutes.
When it is 12:00 in London it is 04:00 in Los Angeles.
At what time, in Los Angeles time, does the flight land? Say if it is on a different day.

Arrival in London time = 14:05 + 12 h 35 min = 02:40 the next day
Los Angeles is 8 hours behind London, so subtract 8 hours
18:40
Answer: 18:40 the same day

3 Reading a trigonometric graph

The diagram shows the graph of $y = a \sin bx° + c$ for $0 \leq x \leq 360$.
a, b and c are integers.
Find the values of a, b and c.

Greatest value = 2, least value = −6
c = ½(2 + (−6)) = −2 and a = ½(2 − (−6)) = 4
One full cycle takes 90° (4 cycles between 0 and 360), so b = 360 ÷ 90 = 4
Answer: a = 4, b = 4, c = −2

4 Algebraic fractions: multiplying and dividing

Simplify fully $\frac{x^2 - 49}{x^2 + x} \div \frac{x^2 - 4x - 21}{x}$

Factorise: $\frac{(x - 7)(x + 7)}{x(x + 1)} \div \frac{(x + 3)(x - 7)}{x}$

Turn the second fraction upside down and multiply: $\frac{(x - 7)(x + 7)}{x(x + 1)} \times \frac{x}{(x + 3)(x - 7)}$

Cancel (x − 7) and x

$= \frac{(x + 7)}{(x + 1)(x + 3)}$

Answer: $\frac{(x + 7)}{(x + 1)(x + 3)}$

5 Coordinates and gradient at the y-intercept

$y = -3x^4 + x^2 + 2x + 7$
The curve crosses the y-axis at the point A.
Work out the coordinates of A and the gradient of the curve at A.

When x = 0, y = 7, so A is (0, 7)

$\frac{dy}{dx} = -12x^3 + 2x + 2$

When x = 0, $\frac{dy}{dx} = 2$

Answer: A = (0, 7), gradient 2

6 Area of a triangle using sine

Triangle ABC has AB = 11 cm and BC = 20 cm.
The area of triangle ABC is $55\sqrt{3}$ cm^2.
Angle ABC is acute.
Work out the size of angle ABC.
Do not use a calculator.

Area = $\frac{1}{2} \times AB \times BC \times \sin B$

$55\sqrt{3} = \frac{1}{2} \times 11 \times 20 \times \sin B$

$\sin B = 55\sqrt{3} \div 110 = \frac{\sqrt{3}}{2}$

$\sin 60° = \frac{\sqrt{3}}{2}$, and B is acute, so B = 60°

Answer: 60°

7 Finding a matrix from MA = B

$A = \begin{pmatrix} -1 & -2 \\ 0 & -1 \end{pmatrix}$ and $B = \begin{pmatrix} 9 & 2 \\ 3 & 2 \end{pmatrix}$.
AM = B, where M is a 2 × 2 matrix.
Find M.

AM = B so $M = A^{-1}B$

$A^{-1} = \begin{pmatrix} -1 & 2 \\ 0 & -1 \end{pmatrix}$

$M = \begin{pmatrix} -1 & 2 \\ 0 & -1 \end{pmatrix}\begin{pmatrix} 9 & 2 \\ 3 & 2 \end{pmatrix} = \begin{pmatrix} -3 & 2 \\ -3 & -2 \end{pmatrix}$

Answer: M = $\begin{pmatrix} \mathbf{-3} & \mathbf{2} \\ \mathbf{-3} & \mathbf{-2} \end{pmatrix}$

8 Histograms with an unmarked scale

The histogram shows the lengths of leaves collected from a tree.
360 of them had a length between 40 and 70 cm.
(a) How many had a length between 20 and 40 cm?
(b) How many leaves are there in total?

Frequency is proportional to the area of a bar
40 to 70: bar is 6 squares high and 30 wide, area 180, which represents 360
So 1 unit of area represents 360 ÷ 180 = 2
(a) 20 to 40: 5 × 20 = 100 units of area, so 100 × 2 = 200
(b) Total area = 40 + 10 + 100 + 180 = 330, so total = 330 × 2 = 660
Answer: (a) 200 (b) 660

9 Greatest speed from displacement

A particle moves in a straight line. After t seconds, its displacement from a fixed point O is s metres, where
$s = -t^3 + 9t^2$ for $t \geq 0$
Find the greatest speed of the particle, and the value of t at which it occurs.

$v = \frac{ds}{dt} = -3t^2 + 18t$

$a = \frac{dv}{dt} = -6t + 18$

Maximum v when a = 0: $-6t + 18 = 0$, so t = 3

$\frac{d^2v}{dt^2} = -6 < 0$, so v is a maximum

v = 27

Answer: 27 m/s when t = 3 s

10 Reverse percentages

The price of a tablet is increased by 10%.
Later the new price is reduced by 40% in a sale.
The sale price is £99.
Work out the price before the increase. (Calculator)

Multipliers: 1.1 and 0.6
Overall multiplier = 1.1 × 0.6 = 0.66
Original × 0.66 = £99
Original = £99 ÷ 0.66 = £150
Answer: £150