Day 168 **IGCSE 10-a-day: Model Solutions**

1 Differentiating polynomials

$y = 6x^3 - 7x^2 - 3$

Find $\frac{dy}{dx}$

Differentiate each term: $x^n \to nx^{n-1}$ and a constant $\to 0$

$6x^3 \to 18x^2$, $-7x^2 \to -14x$

$\frac{dy}{dx} = 18x^2 - 14x$

Answer: $\mathbf{\frac{dy}{dx} = 18x^2 - 14x}$

2 Averages with algebra

Here are five numbers: $2x + 4$, x, $3x + 2$, $x + 6$, $3x - 2$.
The mean of the five numbers is 18.
(a) Work out the value of x.
(b) Work out the median of the five numbers.

(a) Total = $(2x + 4) + (x) + (3x + 2) + (x + 6) + (3x - 2) = 18 \times 5 = 90$
$10x + 10 = 90$, so $10x = 80$ and $x = 8$
The numbers are 20, 8, 26, 14, 22
(b) In order: 8, 14, 20, 22, 26, so the median is 20
Answer: (a) x = 8 (b) 20

3 Velocity and acceleration

A particle P moves in a straight line. After t seconds its displacement from a fixed point O is s metres, where
$s = 2t^3 + 2t^2 + 5t + 1$ $(t > 0)$
Find (a) the velocity of P, (b) the acceleration of P, when t = 3.

$v = \frac{ds}{dt} = 6t^2 + 4t + 5$

$a = \frac{dv}{dt} = \frac{d^2s}{dt^2} = 12t + 4$

When t = 3: v = 71 and a = 40
Answer: (a) 71 m/s (b) 40 m/s^2

4 Perimeter and area of a running track

The diagram shows a running track made from a rectangle and two semicircles. (Calculator)
An athlete runs 5 complete laps around the track.
Work out the total distance run, to the nearest 10 metres.
The diagram is not drawn accurately.

Boundary of one lap = $2 \times 50 + 2 \times \pi \times 20 = 225.66$ m
5 laps = $5 \times 225.66 = 1128.3$ m
= 1130 m to the nearest 10 m
Answer: 1130 m

5 Second derivative and nature at a given point

$y = -2x^3 + 12x^2 - 18x - 2$
Use the second derivative to show that the curve has a minimum at x = 1.

$\frac{dy}{dx} = -6x^2 + 24x - 18$

When x = 1: $\frac{dy}{dx} = 0$, so it is a stationary point

$\frac{d^2y}{dx^2} = -12x + 24$

When x = 1: $\frac{d^2y}{dx^2} = 12$, which is positive

So there is a minimum at x = 1
Answer: minimum at x = 1

6 Direct and inverse proportion

P is inversely proportional to t.
When t = 6, P = 2.
(a) Find a formula for P in terms of t.
(b) Find the value of t when P = 4.

(a) $P = \frac{k}{t}$

Substitute t = 6 and P = 2: $2 = \frac{k}{6}$

k = 12

$P = \frac{12}{t}$

(b) $4 = \frac{12}{t}$, so t = 12 ÷ 4 = 3

Answer: (a) $P = \frac{12}{t}$ (b) t = 3

7 Histogram with unequal class widths

The histogram shows the masses, in kg, of some parcels.
The bar for 10 to 30 represents 6 parcels.
(a) Work out the frequency of the class 80 to 100.
(b) Work out the total frequency.

Frequency = k × height × width. For 10 to 30: 6 = k × 3 × 20 so $k = \frac{1}{10}$

(a) 80 to 100: $\frac{1}{10} \times 2 \times 20 = 4$

(b) Frequencies: 6, 5, 6, 10, 4
Total = 31
Answer: (a) 4 (b) 31

8 Algebraic fractions: multiplying and dividing

Simplify fully $\frac{x^2 - x - 20}{x - 3} \div \frac{x - 5}{x^2 - 4x + 3}$

Factorise: $\frac{(x + 4)(x - 5)}{(x - 3)} \div \frac{(x - 5)}{(x - 3)(x - 1)}$

Turn the second fraction upside down and multiply: $\frac{(x + 4)(x - 5)}{(x - 3)} \times \frac{(x - 3)(x - 1)}{(x - 5)}$

Cancel (x − 5) and (x − 3)
= (x − 1)(x + 4)
Answer: (x − 1)(x + 4)

9 Surds with unknowns

Given that $(a + \sqrt{6})(b + \sqrt{6}) = 51 + 14\sqrt{6}$,
where a and b are positive integers with a < b, find the values of a and b.

$(a + \sqrt{6})(b + \sqrt{6}) = ab + (a + b)\sqrt{6} + 6$
Compare the $\sqrt{6}$ terms: a + b = 14
Compare the integer terms: ab + 6 = 51, so ab = 45
Two numbers with sum 14 and product 45 are 5 and 9
Answer: a = 5, b = 9

10 Probability and quadratic equations

A bag contains x white beads and 5 green beads.
Two beads are taken at random without replacement.

The probability that both are white is $\frac{1}{6}$.

(a) Show that $x^2 - 3x - 4 = 0$.
(b) Hence work out the number of beads in the bag.

(a) P(both white) = $\frac{x}{x+5} \times \frac{x-1}{x+4} = \frac{1}{6}$

$6\,x(x-1) = 1(x+5)(x+4)$

$6x^2 - 6x = x^2 + 9x + 20$ so $5x^2 - 15x - 20 = 0$; divide by 5: $x^2 - 3x - 4 = 0$

(b) Solve: $x = 4$ or $x = -1$

x cannot be negative, so $x = 4$

Number of beads = 4 + 5 = 9

Answer: x = 4, so 9 beads in the bag