Day 167 **IGCSE 10-a-day: Model Solutions**

1 Quadratic sequences

Here are the first five terms of a quadratic sequence.
7, 11, 19, 31, 47
Find an expression for the nth term.

First differences: 4, 8, 12, 16
Second differences: 4, 4, 4, so a = 4 ÷ 2 = 2
Subtract $2n^2$ from each term: 5, 3, 1, −1, −3
The remaining linear sequence has nth term $-2n + 7$
nth term = $2n^2 - 2n + 7$
Answer: $2n^2 - 2n + 7$

2 Normal in the form ax + by + c = 0

The point A lies on the curve $y = -2x^2 - 5x - 1$ where x = 0.
Find the equation of the normal at A. Give your answer in the form ax + by + c = 0 where a, b and c are integers.

y = −1 at x = 0 and gradient of curve = −5
Gradient of normal = $\frac{1}{5}$
$y + 1 = \frac{1}{5}x$
$y = \frac{1}{5}x - 1$
Multiply by 5: $x - 5y - 5 = 0$
Answer: $x - 5y - 5 = 0$

3 Vector between two points

A is the point (−5, 2) and B is the point (8, −1).
(a) Write down the vector **AB**.
(b) Find |**AB**|, giving the answer as a surd in simplest form if it is not exact.

(a) $\mathbf{AB} = \begin{pmatrix} 8 \\ -1 \end{pmatrix} - \begin{pmatrix} -5 \\ 2 \end{pmatrix} = \begin{pmatrix} 13 \\ -3 \end{pmatrix}$
(b) $|\mathbf{AB}| = \sqrt{(13)^2 + (-3)^2} = \sqrt{178}$
Answer: (a) $\begin{pmatrix} 13 \\ -3 \end{pmatrix}$ (b) $\sqrt{178}$

4 Transformations of graphs

The point P(6, 2) lies on the graph of y = f(x).
Write down the coordinates of the image of P on the graph of
(a) y = f(x + 2) (b) y = f(x) + 2

(a) y = f(x + 2) is a translation of 2 to the left: (4, 2)
(b) y = f(x) + 2 is a translation of 2 up: (6, 4)
Answer: (a) (4, 2) (b) (6, 4)

5 Points with a given gradient (cubic)

The curve $y = x^3 + 3x^2 - 18x + 5$ has gradient −9 at two points.
Find the x coordinates of these points.

$\frac{dy}{dx} = 3x^2 + 6x - 18$
$3x^2 + 6x - 18 = -9$, so $3x^2 + 6x - 9 = 0$
Divide by 3: $x^2 + 2x - 3 = 0$
(x + 3)(x − 1) = 0, so x = −3 or x = 1
Answer: x = −3 and x = 1

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6 Simultaneous equations with an inverse matrix

$M = \begin{pmatrix} -2 & -3 \\ -1 & -2 \end{pmatrix}$.

(a) Find M^{-1}.

(b) Use your answer to (a) to solve

$-2x - 3y = -18$

$-x - 2y = -12$

(a) det M = 1 so $M^{-1} = \begin{pmatrix} -2 & 3 \\ 1 & -2 \end{pmatrix}$

(b) The equations are $\begin{pmatrix} -2 & -3 \\ -1 & -2 \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} -18 \\ -12 \end{pmatrix}$

$\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} -2 & 3 \\ 1 & -2 \end{pmatrix}\begin{pmatrix} -18 \\ -12 \end{pmatrix} = \begin{pmatrix} 0 \\ 6 \end{pmatrix}$

Answer: (a) $\begin{pmatrix} \mathbf{-2} & \mathbf{3} \\ \mathbf{1} & \mathbf{-2} \end{pmatrix}$ **(b) x = 0, y = 6**

7 Rate of change of area or volume

The volume, V cm^3, of a sphere of radius r cm is $V = \frac{4}{3}\pi r^3$.

(a) Find $\frac{dV}{dr}$ when r = 6.

(b) Find the value of r when $\frac{dV}{dr} = 324\pi$.

$\frac{dV}{dr} = 4\pi r^2$

When r = 6: $\frac{dV}{dr} = 144\pi$

$4\pi r^2 = 324\pi$, so $4r^2 = 324$ and r = 9 (lengths are positive)

Answer: (a) 144π (b) r = 9

8 Successive percentage change

The price of a laptop is decreased by 50%.
The new amount is then decreased by 20%.
Work out the overall percentage change.

Multipliers: 0.5 and 0.8
0.5 × 0.8 = 0.4
0.4 is a decrease of 60%
Answer: 60% decrease

9 Percentages with an unknown rate

A population is decreased by x% in one year and then decreased by a further x% the next year.
Over the two years the population decreases by 9.75%.
Work out the value of x.

Yearly multiplier = $1 - \frac{x}{100}$

Two years: $(1 - \frac{x}{100})^2 = 0.9025$

Take the square root: $1 - \frac{x}{100} = 0.95$

$\frac{x}{100} = 0.05$, so x = 5

Answer: x = 5

10 Cone made from a sector

A sector of a circle has radius 20 cm and angle 144°.
The sector is made into a cone by joining the two straight edges together.
(a) Work out the radius of the base of the cone.
(b) Work out the exact volume of the cone. Give your answer in terms of π, as a surd if necessary.

(a) Arc length of the sector = $\frac{144}{360} \times 2\pi \times 20 = 16\pi$ cm

This is the circumference of the base: $2\pi r = 16\pi$, so r = 8 cm

(b) Slant height = 20 cm, so height$^2 = 20^2 - 8^2 = 336$, height = $4\sqrt{21}$ cm

Volume = $\frac{1}{3} \times \pi \times 8^2 \times 4\sqrt{21} = \frac{256\sqrt{21}}{3}\pi$ cm^3

Answer: (a) 8 cm (b) $\frac{256\sqrt{21}}{3}\pi$ cm^3