Day 166 **IGCSE 10-a-day: Model Solutions**

1 Differentiating polynomials

Find $\frac{dy}{dx}$ when $y = 6x^4 - 3x^2 - 5x$

Differentiate each term: $x^n \to nx^{n-1}$ and a constant $\to 0$
$6x^4 \to 24x^3$, $-3x^2 \to -6x$, $-5x \to -5$
$\frac{dy}{dx} = 24x^3 - 6x - 5$

Answer: $\mathbf{\frac{dy}{dx} = 24x^3 - 6x - 5}$

2 Equations with fractions

Solve $\frac{x-1}{5} + \frac{x-3}{2} = -1$

Multiply every term by 10: $2(x - 1) + 5(x - 3) = -10$
Expand: $7x - 17 = -10$
$7x = 7$
$x = 1$
Answer: $x = 1$

3 Tests of divisibility

The 5-digit number 37k44 is a multiple of 3.
k is a single digit.
Find all the possible values of k.

A number is a multiple of 3 if its digit sum is a multiple of 3
Digit sum = 3 + 7 + 4 + 4 + k = 18 + k
This must be a multiple of 3, so k = 0 or 3 or 6 or 9
Answer: k = 0 or 3 or 6 or 9

4 Exchange rates

Kira changes £200 into euros at £1 = €1.35.
Kira spends €130 and then changes the rest back into pounds at £1 = €1.6.
How many pounds does Kira get back?

£200 × 1.35 = €270
270 − 130 = €140
140 ÷ 1.6 = £87.50
Answer: £87.50

5 Trigonometric graphs

The diagram shows the graph of $y = a \cos x° + c$ for $0 \leq x \leq 360$.
Some points on the graph are labelled.
Find the value of a and the value of c.

At x = 0, cos x° = 1, so a + c = 5
At x = 180, cos x° = −1, so −a + c = 3
Add: 2c = 8, so c = 4
Subtract: 2a = 2, so a = 1
Answer: a = 1, c = 4

6 Find constants from a gradient

A curve has equation $y = ax^2 + \frac{b}{x}$, where a and b are constants.

The point (1, 5) lies on the curve and the gradient of the curve at this point is 1.
Find the values of a and b.

$\frac{dy}{dx} = 2ax - bx^{-2} = 2ax - \frac{b}{x^2}$

Point on curve: $a + b = 5$. Gradient: $2a - b = 1$
Solving simultaneously gives $a = 2$ and $b = 3$
Answer: a = 2, b = 3

7 Using a quadratic nth term

The nth term of a sequence is $n^2 + 8n - 1$.
(a) Work out the 11th term.
(b) One term of the sequence is 344. Work out the value of n for this term.

(a) $1 \times 11^2 + 8 \times 11 - 1 = 208$
(b) $n^2 + 8n - 1 = 344$ so $n^2 + 8n - 345 = 0$
Solve (factorise or use the formula): $n = 15$ or $n = -23$. n must be a positive integer, so $n = 15$
Answer: (a) 208 (b) n = 15

8 Tangent to a circle

A circle has equation $x^2 + y^2 = 841$.
The point P(−21, 20) lies on the circle.
(a) Find the equation of the tangent to the circle at P. Give your answer in the form ax + by = c, where a, b and c are integers.
(b) The tangent meets the x-axis at the point T. Write down the coordinates of T.

(a) The gradient of the radius OP is $-\frac{20}{21}$

The tangent is perpendicular to the radius, so its gradient is $-1 \div (-\frac{20}{21}) = \frac{21}{20}$

$y - 20 = \frac{21}{20}(x - (-21))$

Multiply out and rearrange: $21x - 20y = -841$
(b) On the x-axis $y = 0$, so $21x = -841$
$x = -\frac{841}{21}$, so T is $(-\frac{841}{21}, 0)$

Answer: (a) $21x - 20y = -841$ (b) T = $(-\frac{841}{21}, 0)$

9 Trapezium with vectors

OABC is a trapezium. OA is parallel to CB and $CB = \frac{1}{4}OA$.

$\mathbf{OA} = \mathbf{a}$ and $\mathbf{OC} = \mathbf{c}$.
(a) Find $\mathbf{AB}$ in terms of $\mathbf{a}$ and $\mathbf{c}$.
(b) AM : MB = 3 : 2. Find $\mathbf{OM}$.

(a) $\mathbf{CB} = \frac{1}{4}\mathbf{a}$

$\mathbf{OB} = \mathbf{OC} + \mathbf{CB} = \frac{1}{4}\mathbf{a} + \mathbf{c}$

$\mathbf{AB} = -\mathbf{OA} + \mathbf{OB} = -\frac{3}{4}\mathbf{a} + \mathbf{c}$

(b) $\mathbf{OM} = \mathbf{OA} + \frac{3}{5}\mathbf{AB} = \mathbf{a} + \frac{3}{5}(-\frac{3}{4}\mathbf{a} + \mathbf{c})$

$= \frac{11}{20}\mathbf{a} + \frac{3}{5}\mathbf{c}$

Answer: (a) $-\frac{3}{4}\mathbf{a} + \mathbf{c}$ (b) $\frac{11}{20}\mathbf{a} + \frac{3}{5}\mathbf{c}$

10 Vertical distance between stationary points

The curve $y = 2x^3 + 9x^2 + 12x + 3$ has two stationary points, A and B.
Work out the vertical distance between A and B.

$\frac{dy}{dx} = 6x^2 + 18x + 12 = 6(x + 2)(x + 1)$

$x = -2$ and $x = -1$

At $x = -2$, $y = -1$; at $x = -1$, $y = -2$

Difference $= -1 - (-2) = 1$

Answer: 1