Day 165 **IGCSE 10-a-day: Model Solutions**

1 Solving trigonometric equations

Solve $\tan x = \sqrt{3}$ for $0° \leq x \leq 360°$.
Do not use a calculator.

The related acute angle satisfies $\tan \theta = \sqrt{3}$, so $\theta = 60°$
tangent is positive in the 1st and 3rd quadrants, so $x = 60°$ and $x = 180° + 60°$
$x = 60°$ and $x = 240°$
Answer: $x = 60°$ or $x = 240°$

2 Expand then differentiate

$y = x(x + 2)(3x - 1)$
Find $\frac{dy}{dx}$.

Expand: $y = 3x^3 + 5x^2 - 2x$
$3x^3 \to 9x^2$, $5x^2 \to 10x$, $-2x \to -2$
$\frac{dy}{dx} = 9x^2 + 10x - 2$
Answer: $\frac{dy}{dx} = 9x^2 + 10x - 2$

3 Position vector of a point dividing a line

The position vectors of A and B are $\begin{pmatrix} -1 \\ 0 \end{pmatrix}$ and $\begin{pmatrix} 9 \\ -5 \end{pmatrix}$.
The point P lies on AB so that AP : PB = 3 : 2.
Find the position vector of P.

$\mathbf{AB} = \begin{pmatrix} 9 \\ -5 \end{pmatrix} - \begin{pmatrix} -1 \\ 0 \end{pmatrix} = \begin{pmatrix} 10 \\ -5 \end{pmatrix}$
$\mathbf{AP} = \frac{3}{5}\mathbf{AB} = \begin{pmatrix} 6 \\ -3 \end{pmatrix}$
$\mathbf{OP} = \mathbf{OA} + \mathbf{AP} = \begin{pmatrix} 5 \\ -3 \end{pmatrix}$
Answer: $\begin{pmatrix} \mathbf{5} \\ \mathbf{-3} \end{pmatrix}$

4 Nested surds

Show that $\sqrt{45 + 20\sqrt{5}} = 5 + 2\sqrt{5}$

Square the right-hand side: $(5 + 2\sqrt{5})^2 = 25 + 20\sqrt{5} + 20$
$= 45 + 20\sqrt{5}$
So $\sqrt{45 + 20\sqrt{5}} = 5 + 2\sqrt{5}$
Answer: $\sqrt{45 + 20\sqrt{5}} = 5 + 2\sqrt{5}$

5 Inclusion and exclusion with two sets

There are 36 people.
17 of them own a cat and 14 of them own a dog.
7 of them do neither.
(a) Work out how many do both.
(b) One of the people is chosen at random. Find the probability that they only own a cat.

Number in A or B = $36 - 7 = 29$
(a) $n(A \cap B) = n(A) + n(B) - n(A \cup B) = 17 + 14 - 29 = 2$
(b) Number who only own a cat = $17 - 2 = 15$
Probability = $\frac{15}{36} = \frac{5}{12}$
Answer: (a) 2 (b) $\frac{5}{12}$

6 Three stationary points of a quartic

$y = x^4 - 18x^2 + 1$

Find the coordinates of the stationary points of the curve and determine the nature of each.

$\frac{dy}{dx} = 4x^3 - 36x = 4x(x - 3)(x + 3)$

$\frac{dy}{dx} = 0$ gives $x = -3$, $x = 0$ or $x = 3$

Points: $(-3, -80)$, $(0, 1)$, $(3, -80)$

$\frac{d^2y}{dx^2} = 12x^2 - 36$

$x = -3$: $\frac{d^2y}{dx^2} = 72$; $x = 0$: $\frac{d^2y}{dx^2} = -36$; $x = 3$: $\frac{d^2y}{dx^2} = 72$

So $(-3, -80)$ is a minimum, $(0, 1)$ is a maximum, $(3, -80)$ is a minimum

Answer: $(-3, -80)$ minimum; $(0, 1)$ maximum; $(3, -80)$ minimum

7 Mixtures and percentage profit

A shop mixes two types of flour.
Type A costs £12.00 per kg and type B costs £10.00 per kg.
They are mixed in the ratio A : B = 2 : 3.
The mixture is sold for £15.12 per kg.
Work out the percentage profit.

Cost of 5 kg of mixture = 2 × £12.00 + 3 × £10.00 = £54.00
Cost per kg = £54.00 ÷ 5 = £10.80
Profit per kg = £15.12 − £10.80 = £4.32
Percentage profit = 4.32 ÷ 10.80 × 100 = 40%

Answer: 40%

8 Increasing or decreasing at a point

$y = -x^3 - 6x^2 - 9x$

At the point where $x = 3$, is the curve increasing or decreasing? You must show your working.

$\frac{dy}{dx} = -3x^2 - 12x - 9$

When $x = 3$: $\frac{dy}{dx} = -72$

-72 is negative, so the curve is decreasing

Answer: decreasing (gradient -72)

9 Simple and compound interest

Mo invests £2,500 for 5 years at 2.5% per year simple interest.
Ivan invests £2,500 for 5 years at 2.5% per year compound interest.
Work out how much more interest Ivan earns than Mo.
Give your answer to the nearest penny. (Calculator)

Simple interest: £2,500 × 2.5% × 5 = £312.50
Compound: £2,500 × 1.025^5 = £2,828.52
Compound interest = £2,828.52 − £2,500 = £328.52
Difference = £328.52 − £312.50 = £16.02

Answer: £16.02

10 Circle and line intersections

Solve the simultaneous equations

$x^2 + y^2 = 74$

$y = x - 12$

Substitute into the first equation: $x^2 + (x - 12)^2 = 74$

Expand: $2x^2 - 24x + 70 = 0$

Divide by 2: $x^2 - 12x + 35 = 0$

Factorise: $(x - 5)(x - 7) = 0$, so $x = 5$ or $x = 7$

$y = x - 12$ gives $y = -7$ and $y = -5$

Answer: $x = 5, y = -7$ and $x = 7, y = -5$