Day 164 **IGCSE 10-a-day: Model Solutions**

1 Gradient at a point

Work out the gradient of the curve $y = -x^4 - 7x^3 + 6$ at the point where $x = 3$.

$\frac{dy}{dx} = -4x^3 - 21x^2$

When $x = 3$: gradient $= -297$

Answer: -297

2 Direct variation

M is directly proportional to the square of n.
When n = 2, M = 12.
(a) Find a formula for M in terms of n.
(b) Find the value of n when M = 300.

$M = kn^2$

$12 = k \times 2^2$ so $k = 3$

$M = 3n^2$

$300 = 3n^2$ so $n^2 = 100$

$n = 10$

Answer: (a) $M = 3n^2$ (b) $n = 10$

3 Percentages

D is 15% more than A.
A is 5% more than B.
Work out D as a percentage of B.

$D = 1.15 \times A$ and $A = 1.05 \times B$

$D = 1.15 \times 1.05 \times B = 1.2075 \times B$

$1.2075 \times 100 = 120.75\%$

Answer: 120.75%

4 Increasing for x greater than k

The function $f(x) = 2x^2 - 12x - 7$ is increasing when $x > k$.
Find the value of k.

$f'(x) = 4x - 12$

The turning point is where $f'(x) = 0$: $4x - 12 = 0$

$x = 3$

$f'(x)$ is positive for $x > 3$, so $k = 3$

Answer: $k = 3$

5 Simplifying surds

Write $\sqrt{108} + \sqrt{75} - \sqrt{12}$ in the form $a\sqrt{3}$
where a is an integer.

$\sqrt{108} = 6\sqrt{3}$; $\sqrt{75} = 5\sqrt{3}$; $\sqrt{12} = 2\sqrt{3}$

$(6 + 5 - 2)\sqrt{3} = 9\sqrt{3}$

Answer: $9\sqrt{3}$

6 Inverse functions

$f(x) = \frac{4x + 2}{x - 5}$

(a) Find $f^{-1}(x)$.

(b) Find the value of $f^{-1}(-18)$.

(a) Let $y = \frac{4x + 2}{x - 5}$. Then $y(x - 5) = 4x + 2$

$xy - 5y = 4x + 2$

Collect x terms: $xy - 4x = 2 + 5y$

Factorise: $x(y - 4) = 5y + 2$, so $x = \frac{5y + 2}{y - 4}$

$f^{-1}(x) = \frac{5x + 2}{x - 4}$

(b) $f^{-1}(-18) = \frac{5 \times (-18) + 2}{1 \times (-18) - 4} = \frac{-88}{-22} = 4$

Answer: (a) $f^{-1}(x) = \frac{5x + 2}{x - 4}$ (b) 4

7 Three sets from information

67 pupils were asked which activities they do.

A: play football B: play netball C: play rugby

$n(A) = 30$, $n(B) = 33$, $n(C) = 35$, $n(A \cap B) = 16$, $n(A \cap C) = 9$, $n(B \cap C) = 18$, $n(A \cap B \cap C) = 4$.

(a) Work out how many are in none of A, B and C.

(b) Work out how many are in exactly one of A, B and C.

$n(A \cup B \cup C) = 30 + 33 + 35 - 16 - 9 - 18 + 4 = 59$

(a) Number in none of the sets $= 67 - 59 = 8$

(b) Only $A = 30 - 16 - 9 + 4 = 9$

Only B = 3, only C = 12, so exactly one $= 9 + 3 + 12 = 24$

Answer: (a) 8 (b) 24

8 Venn diagrams with algebra

The Venn diagram shows how many of 20 students play the piano (P) or the guitar (G).

(a) Work out the value of x.

(b) A student is chosen at random from those who play the guitar. Work out the probability that this student also plays the piano.

(a) Total: $(x + 3) + x + (2x - 1) + 6 = 20$

$4x + 8 = 20$, so $x = 3$

(b) Students in the second circle: $x + 2x - 1 = 3 + 5 = 8$, of whom 3 are in both

Probability $= \frac{3}{8}$

Answer: (a) $x = 3$ (b) $\frac{3}{8}$

9 Radius from chord and height of arc

A circular arc has a chord AB of length 32 cm.

The greatest distance between the chord and the arc is 8 cm.

Work out the radius of the circle.

Let the radius be r. The centre O is on the line through the midpoint M of AB and the highest point of the arc, so $OM = r - 8$

$AM = 32 \div 2 = 16$. Pythagoras in triangle OMA: $r^2 = (r - 8)^2 + 16^2$

$r^2 = r^2 - 16r + 64 + 256$

$16r = 320$, so $r = 20$ cm

Answer: 20 cm

10 Increasing range for y = ax + b/x

$y = 4x + \frac{100}{x}$

Find the values of x for which y is increasing.

$\frac{dy}{dx} = 4 - \frac{100}{x^2}$

$\frac{dy}{dx} > 0$: $4 > \frac{100}{x^2}$

$x^2 > 25$ (x^2 is positive, so multiply through)

$x < -5$ or $x > 5$

Answer: $x < -5$ or $x > 5$