Day 163 **IGCSE 10-a-day: Model Solutions**

1 Comparing costs

FitLife has a joining fee of £5 plus £5.50 per month.
GymNow has a joining fee of £1 plus £6 per month.
(a) Work out the number of months for which the two costs are the same.
(b) Which is cheaper for 13 months, and by how much?

(a) Let n be the number of months. Working in pounds, the costs are equal when $5 + 5.50n = 1 + 6n$
$0.50n = 4$, so $n = 8$
(b) FitLife: £5 + £5.50 × 13 = £76.50
GymNow: £1 + £6 × 13 = £79
FitLife is cheaper by £2.50
Answer: (a) 8 (b) FitLife, by £2.50

2 Where is the second derivative zero?

$y = x^3 - 6x^2 + 7x + 3$

(a) Find $\frac{d^2y}{dx^2}$.

(b) Find the value of x for which $\frac{d^2y}{dx^2} = 0$, and work out the gradient of the curve at this value of x.

$\frac{dy}{dx} = 3x^2 - 12x + 7$

$\frac{d^2y}{dx^2} = 6x - 12$

$\frac{d^2y}{dx^2} = 0$ gives $6x - 12 = 0$, so $x = 2$

$\frac{dy}{dx} = -5$ when $x = 2$

Answer: (a) $6x - 12$ (b) $x = 2$, gradient -5

3 Fraction problems

A bag contains counters.
$\frac{1}{3}$ of the counters are red and $\frac{1}{8}$ of the counters are blue.
The rest of the counters, 26 of them, are green.
How many counters are in the bag?

Fraction that are green $= 1 - \frac{1}{3} - \frac{1}{8} = \frac{13}{24}$

$\frac{13}{24}$ of the total is 26

Total $= 26 \div \frac{13}{24} = 48$

Answer: 48

4 Differentiating brackets with square roots

$y = (\sqrt{x} - 1)(\sqrt{x} + 1)$

Find $\frac{dy}{dx}$. Give each term of your answer in the form ax^n.

Expand: $y = x - 1$
$x \to 1$, $0 \to 0$
$\frac{dy}{dx} = 1$

Answer: $\frac{dy}{dx} = 1$

5 Variation: joint and combined

y is directly proportional to the square of x and inversely proportional to z.
When x = 8 and z = 5, y = 128.
Find y when x = 6 and z = 2.

$y = k\frac{x^2}{z}$

Substitute: 128 = k × 12.8 so k = 10
y = 10 × 18 = 180
Answer: y = 180

6 Nature from the gradient either side

$y = 2x^3 + 12x^2 + 18x - 6$
(a) Show that there is a stationary point at $x = -3$.
(b) By finding the gradient when $x = -\frac{7}{2}$ and when $x = -\frac{5}{2}$, decide whether it is a maximum or a minimum.

$\frac{dy}{dx} = 6x^2 + 24x + 18$, so when $x = -3$: $\frac{dy}{dx} = 0$

$x = -\frac{7}{2}$: gradient = $\frac{15}{2}$ (positive)

$x = -\frac{5}{2}$: gradient = $-\frac{9}{2}$ (negative)

The gradient goes from positive to 0 to negative, so $(-3, -6)$ is a maximum
Answer: (b) maximum

7 3D trigonometry: cuboid

ABCDEFGH is a cuboid. AB = 13 cm, BC = 6 cm and CG = 9 cm.
(a) Work out the length of AG.
(b) Work out the size of angle GAB.
Give answers to 3 significant figures or 1 decimal place. (Calculator)

(a) $AG^2 = 13^2 + 6^2 + 9^2 = 286$, so AG = 16.91 cm
(b) $BG^2 = 6^2 + 9^2 = 117$. AB is perpendicular to the plane BCGF, so angle ABG = 90°

Triangle ABG is right-angled at B, so $\cos GAB = \frac{AB}{AG} = \frac{13}{16.91}$

Angle GAB = 39.8°
Answer: (a) 16.9 cm (b) 39.8°

8 Line of best fit

The scatter graph shows temperature (x) and ice creams sold (y).
The line of best fit passes through (2, 14) and (6, 24).
(a) Find its equation in the form y = mx + c.
(b) Explain what its gradient tells you.

(a) Gradient $m = \frac{24 - 14}{6 - 2} = \frac{10}{4} = 2.5$

y = 2.5x + c: using (2, 14), 14 = 2.5 × 2 + c, so c = 9
y = 2.5x + 9
(b) For each extra degree of temperature, the number of ice creams sold goes up by 2.5
Answer: (a) y = 2.5x + 9 (b) For each extra degree of temperature, the number of ice creams sold goes up by 2.5

9 Vector proof: collinear points

OA = $\mathbf{a}$ and OB = $\mathbf{b}$.
OP = $4\mathbf{a} + 3\mathbf{b}$, OQ = $9\mathbf{a} + 6\mathbf{b}$ and OR = $24\mathbf{a} + 15\mathbf{b}$.
(a) Show that P, Q and R lie on a straight line.
(b) Work out the ratio PQ : QR in its simplest form.

(a) PQ = OQ − OP = $5\mathbf{a} + 3\mathbf{b}$
QR = OR − OQ = $15\mathbf{a} + 9\mathbf{b}$
QR = 3 PQ, so PQ and QR are parallel, and they share the point Q
So P, Q and R lie on a straight line
(b) PQ : QR = 1 : 3
Answer: (b) 1 : 3

10 Direct proportion formulae

E is directly proportional to the square of v.
When v = 3, E = 18.
(a) Find a formula for E in terms of v.
(b) Work out the value of E when v = 10.

(a) $E = kv^2$
$18 = k \times 9$, so $k = 18 \div 9 = 2$
$E = 2v^2$
(b) $E = 2 \times 100 = 200$
Answer: (a) $E = 2v^2$ (b) 200