Day 162 **IGCSE 10-a-day: Model Solutions**

1 Divide then differentiate

$y = \frac{4x^2 - 5x + 7}{x}$

Find $\frac{dy}{dx}$.

Divide each term by x: $y = 4x - 5 + 7x^{-1}$

$4x \to 4$, $7x^{-1} \to -7x^{-2}$

$\frac{dy}{dx} = 4 - 7x^{-2}$

Answer: $\mathbf{\frac{dy}{dx} = 4 - \frac{7}{x^2}}$

2 Geometric sequences

The first three terms of a geometric sequence are
8, 4, 2
(a) Write down the common ratio.
(b) Work out the 6th term.

(a) $4 \div 8 = \frac{1}{2}$

(b) nth term $= 8 \times \left(\frac{1}{2}\right)^{n-1}$

6th term $= 8 \times \left(\frac{1}{2}\right)^5 = \frac{1}{4}$

Answer: (a) $\mathbf{\frac{1}{2}}$ **(b)** $\mathbf{\frac{1}{4}}$

3 Enlargement

Triangle P has vertices A(−3, 1), B(−6, −2) and C(−2, −2).
Triangle Q has vertices A'(−9, −5), B'(−18, −14) and C'(−6, −14).
Q is an enlargement of P.
Describe fully the single transformation that maps P onto Q.

x-step from A to B = −3, x-step from A' to B' = −9
Scale factor = −9 ÷ −3 = 3
The centre C satisfies: C + 3(A − C) = A', so (1 − 3)C = A' − 3 A
x: (−2) × x = (−9) − 3 × (−3) = 0, so x = 0
y: (−2) × y = (−5) − 3 × 1 = −8, so y = 4
Answer: Enlargement, scale factor 3, centre (0, 4)

4 Solve a composite function equation

f(x) = 2x + 3 and $g(x) = x^2 - 1$.
Solve fg(x) = 3.

$fg(x) = 2(x^2 - 1) + 3 = 2x^2 + 1$
$2x^2 + 1 = 3$, so $x^2 = 1$
x = 1 or x = −1
Answer: x = 1 or x = −1

5 Tangents parallel to a line

A curve has equation $y = x^3 - 6x^2 + 8x - 3$.
Find the equations of the two tangents to the curve that are parallel to the line y = 8x + 7.

Parallel lines have equal gradients: gradient = 8
$\frac{dy}{dx} = 3x^2 - 12x + 8 = 8$, so $3x^2 - 12x = 0$
3x(x − 4) = 0 gives x = 0 or x = 4
At x = 0: y = −3, tangent y − (−3) = 8(x − 0)
At x = 4: y = −3, tangent y − (−3) = 8(x − 4)
Answer: y = 8x − 3 and y = 8x − 35

6 Forming and solving quadratic equations

A rectangle has length $(3x + 6)$ cm and width $(x + 3)$ cm.
The area of the rectangle is 126 cm^2.
(a) Show that $3x^2 + 15x - 108 = 0$
(b) Work out the perimeter of the rectangle.

(a) $(3x + 6)(x + 3) = 126$, so $3x^2 + 15x + 18 = 126$
$3x^2 + 15x - 108 = 0$
(b) $3x^2 + 15x - 108 = 0$: $3(x - 4)(x + 9) = 0$
$x = 4$ ($x = -9$ is not possible as lengths are positive)
Length = 18 cm and width = 7 cm, so perimeter = $2 \times (18 + 7) = 50$ cm
Answer: (b) 50 cm

7 Find unknown coefficients from gradients

The curve $y = x^3 + ax^2 + bx$ has gradient 2 when $x = -3$ and gradient -3 when $x = -2$.
Find the values of a and b.

$\frac{dy}{dx} = 3x^2 + 2ax + b$
$x = -3$: $-6a + b = -25$
$x = -2$: $-4a + b = -15$
Subtract and solve: $a = 5$
$b = 5$
Answer: a = 5, b = 5

8 Speed and a moving train

A train travels at a constant speed of 108 km/h.
It takes 20 seconds to go completely through a tunnel which is 210 m long.
Work out the length of the train, in metres.

Speed = 108 km/h = 108,000 ÷ 3600 = 30 m/s
Distance travelled in 20 s = $30 \times 20 = 600$ m
This is the length of the tunnel plus the length of the train
Train = $600 - 210 = 390$ m
Answer: 390 m

9 Algebraic proof (harder)

Prove algebraically that the difference between the squares of any two odd numbers is a multiple of 8.

Let the odd numbers be $2a + 1$ and $2b + 1$, where a and b are integers.
$(2a + 1)^2 - (2b + 1)^2 = 4a^2 + 4a - 4b^2 - 4b = 4(a - b)(a + b + 1)$
$(a - b) + (a + b + 1) = 2a + 1$ is odd, so one of the two brackets is even
So $(a - b)(a + b + 1)$ is even and $4(a - b)(a + b + 1)$ is a multiple of 8
Answer: Proved

10 Nested surds

Show that $\sqrt{79 + 24\sqrt{7}} = 4 + 3\sqrt{7}$

Square the right-hand side: $(4 + 3\sqrt{7})^2 = 16 + 24\sqrt{7} + 63$
$= 79 + 24\sqrt{7}$
So $\sqrt{79 + 24\sqrt{7}} = 4 + 3\sqrt{7}$
Answer: $\sqrt{79 + 24\sqrt{7}} = 4 + 3\sqrt{7}$