Day 161 **IGCSE 10-a-day: Model Solutions**

1 f′(x) notation

f(x) = $3x^2 + 3x + 1$
(a) Find f′(x).
(b) Work out the value of f′(0).

(a) $3x^2 \rightarrow 6x$, $3x \rightarrow 3$
f′(x) = 6x + 3
(b) f′(0) = 3
Answer: (a) f′(x) = 6x + 3 (b) 3

2 Distance-time graphs

The graph shows Maya's cycle ride to a café and back.
(a) Work out Maya's speed on the way back.
(b) Work out Maya's average speed for the whole ride, including the stop.

(a) Return: 15 km in 1 hours, so speed = 15 ÷ 1 = 15 km/h
(b) Total distance = 2 × 15 = 30 km, total time = 3 hours
Average speed = 30 ÷ 3 = 10 km/h
Answer: (a) 15 km/h (b) 10 km/h

3 3D Pythagoras and trigonometry: cuboids

The diagram shows a cuboid ABCDEFGH. (Calculator)
(a) Work out the length of the diagonal AG.
(b) Work out the angle between AG and the base ABCD, to 1 decimal place.
The diagram is not drawn accurately.

(a) $AC^2 = AB^2 + BC^2 = 4^2 + 4^2 = 32$
$AG^2 = AC^2 + CG^2 = 32 + 7^2 = 81$
AG = 9 cm
(b) The angle is GAC. $\tan(GAC) = \frac{CG}{AC} = \frac{7}{\sqrt{32}}$
angle GAC = 51.1° (1 d.p.)
Answer: (a) 9 cm (b) 51.1°

4 Direct variation

A is directly proportional to d.
When d = 10, A = 70.
(a) Find a formula for A in terms of d.
(b) Find A when d = 8.

A = kd
70 = k × 10 so k = 7
A = 7d
A = 7 × 8 = 56
Answer: (a) A = 7d (b) A = 56

5 Bounds in geometry

A rectangle has length 6.9 cm and width 6.5 cm, both correct to 1 decimal place.
Work out the upper bound of the perimeter of the rectangle.

Each length is given to 1 decimal place, so it can be up to 0.05 cm either side
Length is between 6.85 and 6.95, width is between 6.45 and 6.55
Perimeter = 2 × (6.95 + 6.55) = 27 cm
Answer: 27 cm

6 Maximum profit

A company makes and sells x thousand items. All amounts are in thousands of pounds.
The revenue is $R = x(50 - 3x)$ and the cost is $C = 15 + 8x$.
(a) Show that the profit $P = R - C$ is $P = -3x^2 + 42x - 15$.
(b) Find the number of items that must be sold for the profit to be a maximum, and this maximum profit.

$P = 50x - 3x^2 - (15 + 8x) = -3x^2 + 42x - 15$

$\frac{dP}{dx} = -6x + 42 = 0$ gives $x = 7$

$\frac{d^2P}{dx^2} = -6 < 0$, so a maximum

$P = 132$

Answer: 7000 items, maximum profit £132000

7 Quadratic inequalities

Solve $x^2 > 25$

Rearrange: $x^2 - 25 > 0$

Solve $x^2 - 25 = 0$: $(x + 5)(x - 5) = 0$, so $x = -5$ or $x = 5$

The graph is a U shape crossing the x-axis at -5 and 5; it is above the axis outside the roots

$x < -5$ or $x > 5$

Answer: $x < -5$ or $x > 5$

8 Gradient function from a given point

A curve has equation $y = -2x^3 + 6x - 9$.
The point A (0, k) lies on the curve. Work out the value of k and the gradient of the curve at A.

Substitute $x = 0$ into the equation of the curve

$y = -9$ when $x = 0$, so $k = -9$

$\frac{dy}{dx} = -6x^2 + 6$

At A: gradient = 6

Answer: $k = -9$, gradient = 6

9 Intersecting lines in a triangle

$\mathbf{OA} = \mathbf{a}$, $\mathbf{OB} = \mathbf{b}$, $\mathbf{OP} = \frac{2}{3}\mathbf{a}$ and $\mathbf{OQ} = \frac{1}{4}\mathbf{b}$.

AQ and BP meet at X.
(a) Find $\mathbf{OX}$ in terms of $\mathbf{a}$ and $\mathbf{b}$.
(b) Find the ratio AX : XQ.

Let $\mathbf{AX} = s\mathbf{AQ}$. Then $\mathbf{OX} = \mathbf{a} + s(-\mathbf{a} + \frac{1}{4}\mathbf{b}) = (1 - s)\mathbf{a} + \frac{1}{4}s\mathbf{b}$

Let $\mathbf{BX} = t\mathbf{BP}$. Then $\mathbf{OX} = \mathbf{b} + t(\frac{2}{3}\mathbf{a} - \mathbf{b}) = \frac{2}{3}t\mathbf{a} + (1 - t)\mathbf{b}$

Compare coefficients: $1 - s = \frac{2}{3}t$ and $\frac{1}{4}s = 1 - t$

Solving gives $s = \frac{2}{5}$ and $t = \frac{9}{10}$

(a) $\mathbf{OX} = \frac{3}{5}\mathbf{a} + \frac{1}{10}\mathbf{b}$

(b) $AX = \frac{2}{5} \times AQ$, so $AX : XQ = \frac{2}{5} : \frac{3}{5} = 2 : 3$

Answer: (a) $\frac{3}{5}\mathbf{a} + \frac{1}{10}\mathbf{b}$ (b) 2 : 3

Day 161 **IGCSE 10-a-day: Model Solutions**

10 Bounds and accuracy

a = 0.43 and b = 5.06, both correct to 3 significant figures.
Work out a × b.
Write down your answer to a suitable degree of accuracy.
You must show your working and give a reason for your answer. (Calculator)

$0.4295 \leq a < 0.4305$; $5.055 \leq b < 5.065$
Lower bound = 0.4295 × 5.055 = 2.1711225
Upper bound = 0.4305 × 5.065 = 2.1804825
Both bounds round to 2.2 to 2 significant figures, but not to 3 significant figures
Answer: 2.2 (2 s.f.)