Day 160 **IGCSE 10-a-day: Model Solutions**

1 Ordering numbers

Write these numbers in order of size, starting with the largest.

$\frac{2}{3}$, 0.9, 0.54, $\frac{3}{5}$

Convert each to a decimal: $\frac{2}{3} = 0.667$, $0.9 = 0.9$, $0.54 = 0.54$, $\frac{3}{5} = 0.6$

In order: 0.9, $\frac{2}{3}$, $\frac{3}{5}$, 0.54

Answer: 0.9, $\frac{2}{3}$, $\frac{3}{5}$, 0.54

2 Tangent where the curve crosses the y-axis

The curve $y = x^3 + 3x^2 + x - 1$ crosses the y-axis at the point A.
The tangent to the curve at A meets the x-axis at B.
Find the coordinates of B.

At A, $x = 0$ and $y = -1$

$\frac{dy}{dx} = 3x^2 + 6x + 1$, so the gradient at A is 1

Tangent: $y = x - 1$

At B, $y = 0$: $x - 1 = 0$, so $x = 1$

Answer: (1, 0)

3 Writing inequalities from a situation

On a school trip there are x adults and y children.
There are at most 60 people. There are at least 2 children for every adult. There are at least 4 adults.
(a) Write down three inequalities for x and y.
(b) Find the greatest number of children on the school trip.

(a) At most 60 people: $x + y \leq 60$
At least 2 children for every adult: $y \geq 2x$
At least 4 adults: $x \geq 4$
(b) The most children occurs with the fewest adults: $x = 4$
$y \leq 60 - 4 = 56$ and $y \geq 8$, so $y = 56$ is allowed
Answer: (a) $x + y \leq 60$, $y \geq 2x$, $x \geq 4$ (b) 56

4 Particle at rest

A particle P moves in a straight line. After t seconds, its displacement from a fixed point O is s metres, where
$s = t^3 - 12t^2 + 45t + 4$ $(t \geq 0)$
Find the distance between the two positions where P is at rest.

$v = \frac{ds}{dt} = 3t^2 - 24t + 45$

P is at rest when $v = 0$: $3t^2 - 24t + 45 = 0$
$3(t - 3)(t - 5) = 0$, so $t = 3$ or $t = 5$
$s(3) = 58$ and $s(5) = 54$
Distance $= |54 - 58| = 4$
Answer: 4 m

5 At least one

The probability that a bus is more than 5 minutes late on any morning is $\frac{3}{10}$.

Events on different occasions are independent.
Work out the smallest number of mornings needed so that the probability of this happening at least once is greater than 0.95. (Calculator)

P(at least once in n mornings) = $1 - \frac{7}{10}^n$

n = 8: $1 - \frac{7}{10}^8 = 0.9424$

n = 9: $1 - \frac{7}{10}^9 = 0.9596$

0.9596 is the first value above 0.95

Answer: 9

6 Algebraic proof

Show algebraically that $(m + 5)^2 - (m - 5)^2$ is a multiple of 20 for all integer values of m.

$(m + 5)^2 = m^2 + 10m + 25$ and $(m - 5)^2 = m^2 - 10m + 25$

Subtract: $(m + 5)^2 - (m - 5)^2 = 20m$

$20m = 20 \times m$, which is a multiple of 20 because m is an integer

Answer: Proved

7 Exact trigonometry in triangles

ABC is a right-angled triangle.
Work out the exact value of x. Give your answer in its simplest form.
The diagram is not drawn accurately.

x is the opposite side BC and the known side is the adjacent side AB, so use tan 45°

tan 45° = 1

$x = 2\sqrt{2} \times \tan 45° = 2\sqrt{2} \times 1$

$x = 2\sqrt{2}$

Answer: $x = 2\sqrt{2}$

8 Tangent meets the curve again

A curve has equation $y = x^3 - x^2 - 2x - 4$.
The tangent to the curve at the point A, where x = 2, meets the curve again at the point B.
Find the coordinates of B.

$\frac{dy}{dx} = 3x^2 - 2x - 2$, so the gradient at A is 6, and A = (2, −4)

Tangent: $y = 6x - 16$

Equate: $x^3 - x^2 - 8x + 12 = 0$, which has a repeated root x = 2

Sum of the roots = 1, so 2 + 2 + x = 1 and x = −3

y = −34

Answer: B (−3, −34)

9 Show that: consecutive terms of a quadratic sequence

The nth term of a sequence is $2n^2 - 2n - 3$.
(a) Show that the difference between the (n + 1)th term and the nth term is 4n.
(b) Two consecutive terms of the sequence differ by 72. Find the value of n for the first of these two terms.

(a) (n + 1)th term $= 2(n + 1)^2 - 2(n + 1) - 3 = 2n^2 + 2n - 3$

Subtract the nth term: $2n^2 + 2n - 3 - (2n^2 - 2n - 3) = 4n$

(b) 4n = 72 so 4n = 72

n = 18

Answer: (b) n = 18

10 Joint and inverse variation

F is directly proportional to the square of m and inversely proportional to the square of d.
When m = 6 and d = 3, F = 48.
Work out the value of F when m = 2 and d = 2.

$F = \frac{km^2}{d^2}$

$48 = \frac{k \times 36}{9}$, so $k = 48 \times 9 \div 36 = 12$

$F = \frac{12 \times 4}{4} = 12$

Answer: F = 12