Day 16 **IGCSE 10-a-day: Model Solutions**

1 Time zones

A flight leaves London at 09:45 (London time).
It takes 5 hours 25 minutes.
When it is 12:00 in London it is 19:00 in Singapore.
At what time, in Singapore time, does the flight land? Say if it is on a different day.

Arrival in London time = 09:45 + 5 h 25 min = 15:10
Singapore is 7 hours ahead of London, so add 7 hours
22:10
Answer: 22:10 the same day

2 Area of a triangle using sine

Triangle ABC has AB = 16 cm and BC = 15 cm.
The area of triangle ABC is 60 cm^2.
Angle ABC is obtuse.
Work out the size of angle ABC.
Do not use a calculator.

Area = $\frac{1}{2} \times AB \times BC \times \sin B$

$60 = \frac{1}{2} \times 16 \times 15 \times \sin B$

$\sin B = 60 \div 120 = \frac{1}{2}$

$\sin 30° = \frac{1}{2}$, and B is obtuse, so $B = 180° - 30° = 150°$

Answer: 150°

3 Square roots as powers

$y = -4x^{5/2} - x - \frac{6}{x^{3/2}}$

Find $\frac{dy}{dx}$. Give each term of your answer in the form ax^n.

Write as powers of x: $y = -4x^{5/2} - x - 6x^{-3/2}$
$-4x^{5/2} \rightarrow -10x^{3/2}$, $-x \rightarrow -1$, $-6x^{-3/2} \rightarrow 9x^{-5/2}$
$\frac{dy}{dx} = -10x^{3/2} - 1 + 9x^{-5/2}$

Answer: $\frac{dy}{dx} = -10x^{3/2} - 1 + 9x^{-5/2}$

4 Exponential model y = ab^t

$P = ab^t$, where a and b are positive constants.
When t = 1, P = 5400. When t = 3, P = 12150.
Find the values of a and b, and the value of P when t = 5.

Divide: $b^2 = 12150 \div 5400 = 2.25$
b = 1.5
$a \times 1.5^1 = 5400$ so a = 3600
$P = 3600 \times 1.5^5 = 27337.5$
Answer: a = 3600, b = 1.5, P = 27337.5

5 Direct and inverse proportion

V is inversely proportional to the square of r.
When r = 3, V = 1.
(a) Find a formula for V in terms of r.
(b) Find the value of V when r = 1.

(a) $V = \frac{k}{r^2}$

Substitute r = 3 and V = 1: $1 = \frac{k}{3^2}$

k = 9

$V = \frac{9}{r^2}$

(b) $V = \frac{9}{1^2} = 9$

Answer: (a) $V = \frac{9}{r^2}$ (b) 9

6 Local maximum and minimum values

$f(x) = x^3 - 27x + 7$
Find the y coordinate of the local maximum point and the y coordinate of the local minimum point.

$f'(x) = 3x^2 - 27 = 3(x - 3)(x + 3)$
$f'(x) = 0$ gives $x = -3$ or $x = 3$
$f''(x) = 6x$: $f''(-3) = -18 < 0$ (maximum) and $f''(3) = 18 > 0$ (minimum)
$f(-3) = 61$ and $f(3) = -47$
Answer: Local maximum y = 61, local minimum y = −47

7 Quadratic graphs

The curve $y = -x^2 + 14x - 24$ crosses the x-axis at the points A and B.
(a) Find the coordinates of A and B.
(b) Find the coordinates of the maximum point of the curve.

(a) Put y = 0. $-x^2 + 14x - 24 = 0$, so $-(x^2 + 14x + 24) = 0$, giving $-(x - 2)(x - 12) = 0$
$x - 2 = 0$ or $x - 12 = 0$, so x = 2 or x = 12
(b) The turning point is halfway between A and B: $x = (2 + 12) \div 2 = 7$
When x = 7: $y = -1 \times 7^2 + 14 \times 7 - 24 = 25$
Answer: (a) A(2, 0), B(12, 0) (b) (7, 25)

8 Finding a matrix from MA = B

$A = \begin{pmatrix} 2 & -1 \\ 3 & -2 \end{pmatrix}$ and $B = \begin{pmatrix} -3 & -4 \\ -3 & -6 \end{pmatrix}$.
AM = B, where M is a 2 × 2 matrix.
Find M.

AM = B so $M = A^{-1}B$

$A^{-1} = \begin{pmatrix} 2 & -1 \\ 3 & -2 \end{pmatrix}$

$M = \begin{pmatrix} 2 & -1 \\ 3 & -2 \end{pmatrix}\begin{pmatrix} -3 & -4 \\ -3 & -6 \end{pmatrix} = \begin{pmatrix} -3 & -2 \\ -3 & 0 \end{pmatrix}$

Answer: M = $\begin{pmatrix} -3 & -2 \\ -3 & 0 \end{pmatrix}$

9 Tree diagrams and reverse probability

Machine A makes 50% of the bolts in a factory. Machine B makes the rest.
60% of the bolts from machine A are faulty.
10% of the bolts from machine B are faulty.
(a) Work out the probability that a bolt chosen at random is faulty.
(b) A faulty bolt is found. Work out the probability that it was made by machine A.

(a) P(faulty) = 0.5 × 0.6 + 0.5 × 0.1 = 0.3 + 0.05 = 0.35
(b) P(A and faulty) = 0.5 × 0.6 = 0.3

P(A | faulty) = 0.3 ÷ 0.35 = $\frac{6}{7}$

Answer: (a) 0.35 (b) $\frac{6}{7}$

10 Tangent meets the axes

The diagram shows the curve $y = -x^2 + x + 5$ and the tangent at P(1, 5), meeting the axes at A and B.
(a) Find an equation of the tangent at P.
(b) Work out the area of triangle OAB.

$\frac{dy}{dx} = -2x + 1$, so at x = 1 the gradient is −1
$y - 5 = -1(x - 1)$, so $y = -x + 6$
B: x = 0 gives y = 6. A: y = 0 gives x = 6
Area = ½ × 6 × 6 = 18
Answer: (a) $y = -x + 6$ (b) 18